Three Dimensional Trajectories of Interacting Incoherent Photorefractive Solitons

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We investigate theoretically and experimentally the three dimensional propagation and interaction of mutually incoherent screening spatial solitons in an SBN crystal. We show that the interaction of solitons results in complex trajectories which typically show partial mutual spiraling, followed by damped oscillations and the fusion of solitons. This nontrivial behavior is caused by the anisotropy of the nonlinear refractive index change in the crystal. [S0031-9007(98)08243-X]

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The subject of spatial optical solitons exploded in recent years. Spatial solitons have been suggested as the building blocks of all-optical circuitry, thanks to their ability to guide additional beams in a soliton-induced waveguide. This property was used to demonstrate experimentally the soliton-based Y and X waveguides [1–4]. The creation of spatial solitons in a Kerr-type material with an electronic-type nonlinearity requires at least tens of mW of laser light power [4–6]. It was realized lately that the spatial solitons can be created with microwatt laser power in photorefractive (PR) materials. An optical beam propagating through a PR crystal biased with a dc electric field excites charge carriers, which drift and are retrapped on impurity centers. This results in the buildup of a space charge field which screens out the externally applied field. The spatially varying electric field increases the index of refraction in the illuminated region, thereby promoting self-focusing and producing the so-called screening spatial solitons [7–10]. The simplicity of realization as well as extremely low optical power required make the screening solitons attractive for practical applications. Additionally, the screening solitons present a useful tool in experimental verification of theoretical predictions regarding generic properties of individual solitons, as well as soliton pairs. Indeed, recent experiments with the PR strontium barium niobate (SBN) crystal have led to the demonstration of such effects as the spiraling [11], fusion [12,13], birth [14], and annihilation [15] of spatial solitons.

Some time ago it was predicted [16] that the incoherent spatial solitons in a self-focusing medium should spiral around each other if their mutual attraction can counterbalance the divergence of trajectories. A report on the experimental observation of this effect in a PR crystal has been published recently [11]. Mutual rotation for up to 540° over the propagation distance of 13 mm has been reported. For the spiraling to occur it is necessary to have attractive interaction between solitons. While this certainly is the case in isotropic self-focusing Kerr-type materials, the situation with PR media is more complex. As shown not long ago, the nonlinear response of a PR crystal can be strongly anisotropic [17,18]. Two incoherent solitons may experience both attractive and repulsive forces, depending on their relative separation and the location in the crystal [19,20]. Consequently, the resulting trajectories may acquire a more complicated appearance than the simple spiral. We investigate three dimensional (3D) topology of the trajectories of incoherent solitons interacting in PR media. Both the numerical simulation and the experimental observation in the crystal are employed. Numerical simulation is important, as it allows for the probing of details within the crystal, which is not possible in experiment. We find that the generic picture of soliton propagation includes initial spiraling, followed by the oscillation perpendicular to the direction of the applied field, and finally the fusion of beams.

The model describing the interaction of screening solitons in PR media is based on the Kukhtarev material equations and the paraxial approximation to the propagation of optical beam [17,20]. The propagation of beams along the z axis, with an electric field applied along the x axis, and the formation of space charge field are described by the following equations:

$$\left[\frac{\partial}{\partial z} + \vec{\theta}_1 \cdot \nabla - \frac{i}{2}\nabla^2\right]A_1 = \frac{i\gamma}{2}\left(\frac{\partial\varphi}{\partial x} - E_0\right)A_1, \tag{1a}$$

$$\left[\frac{\partial}{\partial z} + \vec{\theta}_2 \cdot \nabla - \frac{i}{2}\nabla^2\right]A_2 = \frac{i\gamma}{2}\left(\frac{\partial\varphi}{\partial x} - E_0\right)A_2, \tag{1b}$$

$$\tau\frac{\partial}{\partial t}(\nabla^2\varphi) + \nabla^2\varphi + \nabla\varphi \cdot \nabla \ln I = E_0\frac{\partial}{\partial x}\ln I + \frac{k_B T}{e}[\nabla^2 \ln I + (\nabla \ln I)^2], \tag{1c}$$

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where $A_1(\vec{r})$ and $A_2(\vec{r})$ are the slowly varying envelopes of the two beams, $\vec{\theta}_1$ and $\vec{\theta}_2$ specify the directions of beam launching, ∇ is the transverse gradient, and $\gamma = k^2 n^4 x_0^2 r_{\rm eff}$ is the medium-light coupling constant. Here k is the wave number of light, n is the index of refraction, x_0 is the typical beam spot size, and $r_{\rm eff}$ is the effective element of the electro-optic tensor. The transverse coordinates x and y are scaled by x_0 and the propagation coordinate z is scaled by the diffraction length $L_D = knx_0^2$. φ is the electrostatic potential induced by the light, with the boundary condition $\nabla\varphi(\vec{r} \to \infty) \to 0$, τ is the relaxation time of the crystal, and E_0 is the external field. The total intensity $I = 1 + |A_1|^2 + |A_2|^2$ is measured in units of the saturation intensity. The last term on the right side of Eq. (1c) describes the diffusion of charges in the crystal. It causes beams to bend.

The set of equations (1) is integrated numerically to steady state, for a range of initial conditions. All material parameters correspond to typical values encountered in SBN crystals [11,19]. The influence of the diffusion field is suppressed by taking the absolute temperature $T = 0$. In experiment this influence is minimized by using relatively wide beams (>20 μm in diameter). In all simulations the input beams are assumed to be Gaussian of sufficient intensity ($I_i = \sim 2-5$) and shape to yield solitons. The trajectory of a soliton is defined as the spatial expectation value of its transverse coordinates,

$$\langle x\rangle(z) = I_{\rm tot}^{-1}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\, x|A(x, y, z)|^2, \quad \text{(2a)}$$

$$\langle y\rangle(z) = I_{\rm tot}^{-1}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\, y|A(x, y, z)|^2. \quad \text{(2b)}$$

These quantities are normalized by the total power of the beam $I_{\rm tot} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(x, y)|^2\, dx\, dy$. This center-of-beam representation is more appropriate than just determining the point of maximum intensity of the beam. It is a good approximation to the beam position even when the deformation of beams is large and the beams split. When the beams propagate close for long distances, they spread and mix and loose the original shape; however, their trajectories are still well defined.

Incoherent PR screening solitons propagating in the plane perpendicular to the direction of the applied field always attract [20]. This attraction leads to the intersection of soliton trajectories in a periodic fashion. The oscillation of solitons is damped and they eventually fuse. The likely reason for damping is the nonintegrability of equations describing soliton interaction, which is reflected in the appearance of radiation losses. The collision of screening solitons is inelastic, hence the possibility of soliton fusion. The launching of incoherent solitons along the direction of applied field leads to their anomalous interaction [19]. Owing to the anisotropy of screening, the soliton interaction is repulsive for well-separated beams and attractive for the overlapping beams. As a consequence, there exist domains of attraction and repulsion in the transverse plane which lead to the nontrivial topology of soliton trajectories when the beams are launched slanted to the direction of external field.

An example of such a complex soliton interaction is presented in Fig. 1. Five separate soliton pairs are launched perpendicularly to the transverse plane, oriented at some angle to the direction of the biasing field. The origin is chosen to be the "center of mass" of all pairs. The pairs initially rotate counterclockwise trying to align along the y axis, as this is the direction of strongest attraction. The momentum acquired by solitons produces an overshoot and, as the beams cross the y axis, the anisotropy of screening slows down the rate of rotation and reverses its direction. The distance between solitons decreases and the pairs twist and turn about the z axis in a damped motion. When viewed along the z axis the motion initially resembles spiraling followed by oscillations predominantly along the y direction. The whole process ends with the soliton fusion. The distance where the beams fuse might be much larger than the typical thickness of the crystal. If the beams are launched anywhere outside the region of attraction they repel and fly apart.

We stress the fact that these trajectories are the result of the interaction of solitons which are initially coplanar. Such behavior does not occur in a typical self-focusing material and is unique to the PR nonlinearity.

Generic examples of the motion of soliton pairs launched obliquely to the direction of the external field are shown in Fig. 2. The pairs 1, 4, and 5 from the spaghetti of solitons in Fig. 1 are untangled and presented

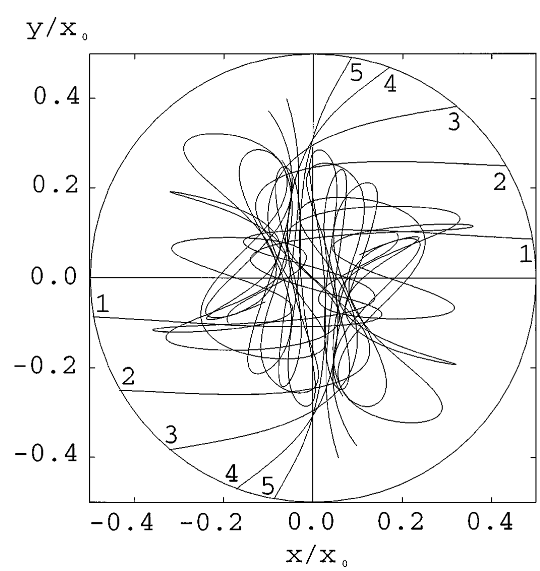

FIG. 1. Projection in the transverse plane of the trajectories of five soliton pairs launched separately at different initial positions within the attractive domain. All pairs propagated for 13 mm.

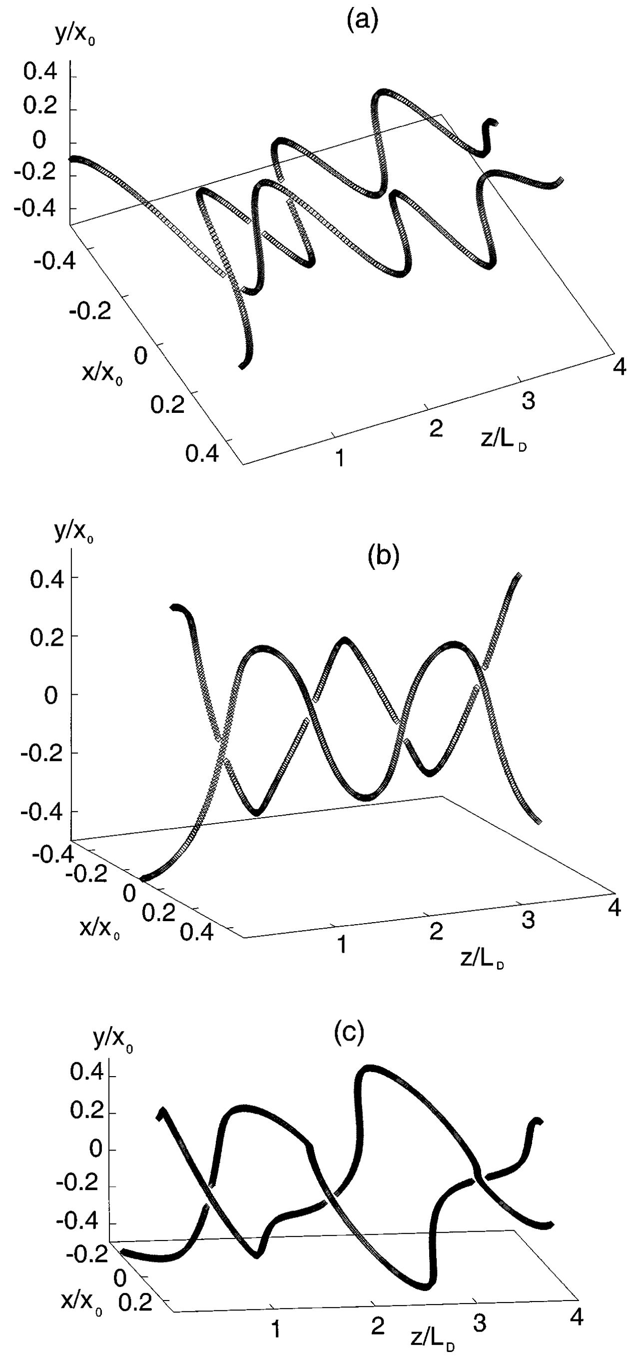

FIG. 2. Pairs 1, 5, and 4 from Fig. 1, shown in 3D. (a) Rotating solitons. (b) Oscillating solitons. (c) Spiraling solitons.

separately. The initial conditions for the pair 4 are slightly changed, to display the prolonged spiraling. It is launched with a small tilt ($\theta_x = \mp 0.55$, $\theta_y = \mp 0.05$) and at a higher intensity ($I_1 = I_2 = 5.2$). The pair 1, shown in Fig. 2(a), starts to rotate; however, after one twist the beams partially unwind and remain on the same side of each other, producing damped oscillations. The pair 5 [Fig. 2(b)] performs elongated oscillations around the y axis from the beginning, wobbling along the way. The pair 4 [Fig. 2(c)] spirals for the whole length, with a swing about the y axis each time it is crossed.

The launching of skewed beams at a higher intensity offers the possibility of prolonged spiraling. Tilted beams carry initial angular momentum relative to the origin. However, the trajectory observed is not a simple, smooth spiral. The beams oscillate while spiraling. The "potential" in which the solitons rotate is not central. The long attractive well along the y axis always breaks the symmetry and prevents indefinite spiraling. In the end, the solitons either fuse or fly apart. It should be mentioned that when the solitons are close to each other and interact strongly, they entangle and their individual identities are rather dubious. The light intensity distributions do not show two distinct beams anymore. However, as soon as the beams disentangle, two bright spots reappear.

The interaction of a pair of mutually incoherent spatial solitons was also investigated experimentally using an SBN crystal measuring $5 \times 6 \times 10$ mm ($c \times b \times a$), and doped with Ce (0.002% by weight). The experimental arrangement was analogous to that employed recently in a study of soliton collisions [15]. Two circular beams (2 μW each) derived from an argon ion laser ($\lambda = 514.5$ nm) were directed by a system of mirrors and beam splitters such that they were focused with Gaussian diameters of 15 μm on the entrance face of the crystal, and were polarized along the c axis, to make use of the r_{33} electro-optic coefficient, which had a measured value of 180 pM/V. A voltage of 2 kV was applied along the c axis and the beams were launched along the a axis. Both beams were made incoherent by reflecting one of them from a mirror mounted on a piezoelectric transducer driven by an ac voltage at several kHz. A white light source was used to control the degree of saturation, so that typically $|A_i|^2 \sim 2$. The input and output light intensity distributions were recorded with a CCD camera. Experimental results are displayed in Figs. 3 and 4, where we show the input and output light intensity distributions for various initial separations and inclinations of the beams.

At first we oriented input beams in such a way that without interaction they would propagate in parallel

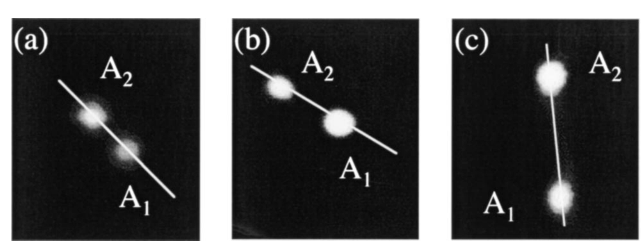

FIG. 3. Mutual rotation of initially coplanar solitons. (a) Initial positions of the beams; (b) relative position of the solitons at the exit face of the crystal for individual propagation; (c) exit position of the solitons for joint propagation. The propagation distance was 10 mm.

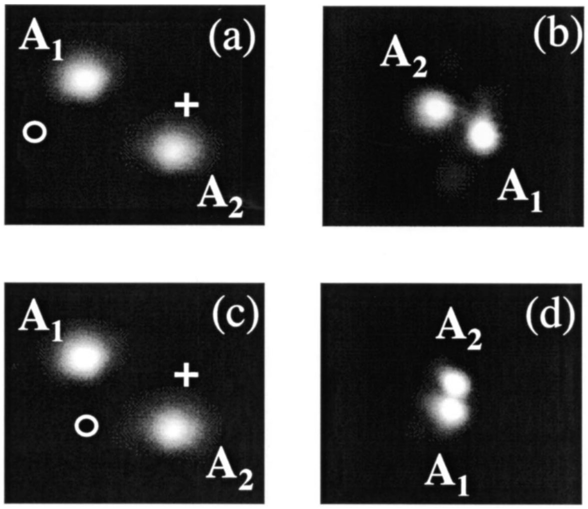

FIG. 4. Spiraling of solitons with initially skewed trajectories. (a),(c) Initial position of the beams; cross and circle denote output positions of solitons A_1 and A_2, respectively, during individual propagation. (b),(d) Output positions of the solitons during simultaneous propagation.

fashion in a plane slanted to the direction of the c axis. For this orientation the strong anisotropy of the electrostatic potential created by both beams results in the interplay of attraction and repulsion along the y and x axes. The net result is the rotation of solitons. Figure 3 depicts mutual clockwise rotation by some 40°. Flipping the initial positions of beams with respect to the vertical plane changes the direction of rotation. This is a consequence of the fact that the solitons always try to align along the y axis.

Figure 4 illustrates soliton interaction for initially tilted trajectories, for two different inclination angles. Figures 4(a) and 4(c) display initial locations of the beams. By a cross (circle) we denoted the output position of the soliton A_1 (A_2) during individual propagation. In both cases the initial trajectories do not intersect. Figures 4(b) and 4(d) display the output positions of both beams at the exit face of the crystal during simultaneous propagation. It is clear that the attraction of solitons resulted in the spiraling of trajectories. Evidently, the rate of rotation depends on the separation of beams and their propagation direction. In the first instance (Fig. 4, top row) the initial trajectories strongly diverge and the soliton attraction is not strong enough to generate substantial rotation. When the divergence of trajectories is decreased, as in the case shown in the bottom row of Fig. 4, the solitons rotate appreciably, by more than 180°. This angle is still considerably less than the angle reported in Ref. [11]. The difference between the two experiments is that the beams in Ref. [11] are of a higher intensity and are launched at a higher initial tilt, which seem to offer the possibility of prolonged spiraling.

In summary, we investigated the 3D interaction of screening solitons in a PR medium. Both the numerical simulation of a theoretical model and the experiment with an SBN crystal have been performed. The off-axes launching in combination with the initial angular tilt makes possible the initial spiraling and oscillatory interaction behavior. The beams perform complicated motion in the transverse plane, rotating about the origin, twisting and turning in the region of attraction; finally, they fuse. We find that the interaction between incoherent screening solitons is basically anisotropic, which may not be apparent in the early stages of soliton propagation.

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