Spatial Screening Solitons as Particles

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Photorefractive spatial screening solitons are treated as rays using geometrical optics. The ray picture is transformed into a classical mechanics picture, in which solitons move self-consistently as particles in a potential created by the induced change in the refractive index. The Hamiltonian equations of motion are integrated to yield trajectories that agree with the optical center-of-mass trajectories. The motion in the transverse plane is found to be not central and the orbits are not closed, preventing the spiraling of solitons.

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Much attention is focused on the recently demonstrated photorefractive (PR) spatial screening solitons. They are generated when a light beam of appropriate wavelength, intensity, and shape [1–4] is launched into a PR crystal and a dc electric field applied in the lateral direction, to induce self-focusing via PR screening. Solitons emerge as a result of the self-induced change in the refractive index, caused by the propagating light beams. Interest in spatial optical solitons stems from their considerable applicative potential [5–8] as well as from their strange behavior [9–13].

The PR effect allows for the self-trapping in one (1D) and two (2D) transverse dimensions at the very low optical power levels (microwatts) [1]. Both 1D and 2D spatial solitons have a unique shape, which is determined by the intensity, the strength of the external field, and the intensity of the background illumination. In addition, incoherent 2D solitons display anomalous interaction behavior [14] due to the anisotropy of self-focusing in 2D, induced by the external field. Anomalous interaction causes the repulsion of incoherent solitons in the direction of applied field, which normally is not observed in Kerr-type materials, and exerts profound influence on the propagation of beams. A proper analysis of screening solitons requires three spatial dimensions and time.

Spatial solitons in PR media do not satisfy the mathematical definition of solitons, even when they propagate as solitary waves with an unchanging beam profile. The term spatial soliton is used in a broader sense to describe nondiffracting self-trapped laser beams. It is well known that solitons in integrable systems behave as particles. In PR materials spatial solitons interact inelastically. Equations describing soliton interaction are nonintegrable, which leads to the radiative and absorptive losses, and damped motion. Nonetheless, an interesting question to ask is whether it is possible to formulate a quasiparticle theory of PR spatial screening solitons. This question is addressed in this Letter [15].

The short answer to the question posed is—yes. However, there are some reservations. The answer is an unequivocal yes only for the well-defined single solitons and well-separated pairs. For the strongly interacting overlapping solitons the answer is maybe. The problem is that the strongly interacting beams deform strongly. They spread, and the identity of individual solitons is questionable. The soliton trajectory, although defined at all times, loses its obvious meaning.

The trajectory of the soliton is defined as the spatial expectation value of its transverse coordinates [10,16] weighted by the transverse intensity,

$$\langle x\rangle(z) = \frac{1}{I_t}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\, x|A(x,y,z)|^2, \qquad (1a)$$

$$\langle y\rangle(z) = \frac{1}{I_t}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, y|A(x,y,z)|^2, \qquad (1b)$$

where $A(x,y,z)$ is the slowly varying envelope of the beam, and I_t is the total transverse intensity, $I_t = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(x,y,z)|^2 dx\, dy$, which acts as an effective mass. The slowly varying envelope obeys the paraxial wave equation for the beam, which is of the form

$$2ikn_0 \frac{\partial A}{\partial z} + \nabla^2 A = -k^2(n^2 - n_0^2)A\,. \qquad (2)$$

The beam propagates in the z direction, with the vacuum wave number k. The operator ∇ is the transverse gradient, n_0 is the unperturbed refractive index of the crystal, and $n^2 = n^2(|A(x,y,z)|^2)$ is (the square of) the refractive index, changed by the propagating beam. The change in the refractive index is an all-important quantity. On one hand, it is proportional to the space-charge field generated by the PR effect; on the other hand, it creates the quasiparticle potential in which the soliton moves.

According to the theory of the PR effect

$$n^2 - n_0^2 = n_0^4 r_{\mathrm{eff}} E_{\mathrm{sc}}\,, \qquad (3)$$

where r_{eff} is the effective component of the electro-optic tensor, and E_{sc} is the space-charge electric field, produced by the redistribution of charges in the crystal. The space-charge field is obtained from the Kukhtarev equations for the PR effect,

$$\nabla \cdot \vec{E}_{\mathrm{sc}} + \vec{E}_{\mathrm{sc}} \cdot \nabla \ln I = -\frac{k_B T}{e}[\nabla^2 \ln I + (\nabla \ln I)^2], \tag{4}$$

where $I = 1 + |A|^2$ is the total light intensity (measured in units of the saturation intensity), k_B is Boltzmann's constant, T is the temperature, and e the electronic charge. In this manner the system of Eqs. (2) and (4) is closed. Numerical treatment of this system is a formidable problem. It is addressed in our other publications [16]. The aim of this paper is to introduce a geometrical optics approach, which offers an alternative, mechanical interpretation and leads to improved understanding.

The basic law of geometrical optics is Fermat's principle, according to which the variation of the optical path between points P and Q in a medium with the index of refraction n is zero,

$$\delta \int_P^Q n \, ds = 0. \tag{5}$$

Since the line element is $ds = (1 + \dot{x}^2 + \dot{y}^2)^{1/2} dz$, one immediately obtains the optical Lagrangian

$$\mathcal{L} = n(1 + \dot{x}^2 + \dot{y}^2)^{1/2}, \tag{6}$$

and can write the Lagrangian equation

$$\frac{d}{ds}\left(n \frac{d\vec{r}}{ds}\right) = \nabla n, \tag{7}$$

which is known as the ray equation. The vector $\vec{r} = (x, y, z)$ denotes the position along the ray propagating from P to Q. The dot in Eq. (6) indicates the derivative with respect to z, which plays the role of time. Using the rules of classical mechanics, one finds the optical Hamiltonian

$$\mathcal{H} = -(n^2 - p^2 - q^2)^{1/2}, \tag{8}$$

where the momenta

$$p = n \frac{dx}{ds}, \qquad q = n \frac{dy}{ds} \tag{9}$$

are the optical direction cosines of the ray [17]. In the paraxial approximation the direction cosines are small. The derivatives with respect to s then become the derivatives with respect to z. The Lagrangian and Hamiltonian functions become

$$\mathcal{L} = \frac{n}{2}(\dot{x}^2 + \dot{y}^2) + n,$$

$$\mathcal{H} = \frac{1}{2n}(p^2 + q^2) - n, \tag{10}$$

and the ray propagation is governed by the corresponding equations of motion. A mechanical interpretation is that

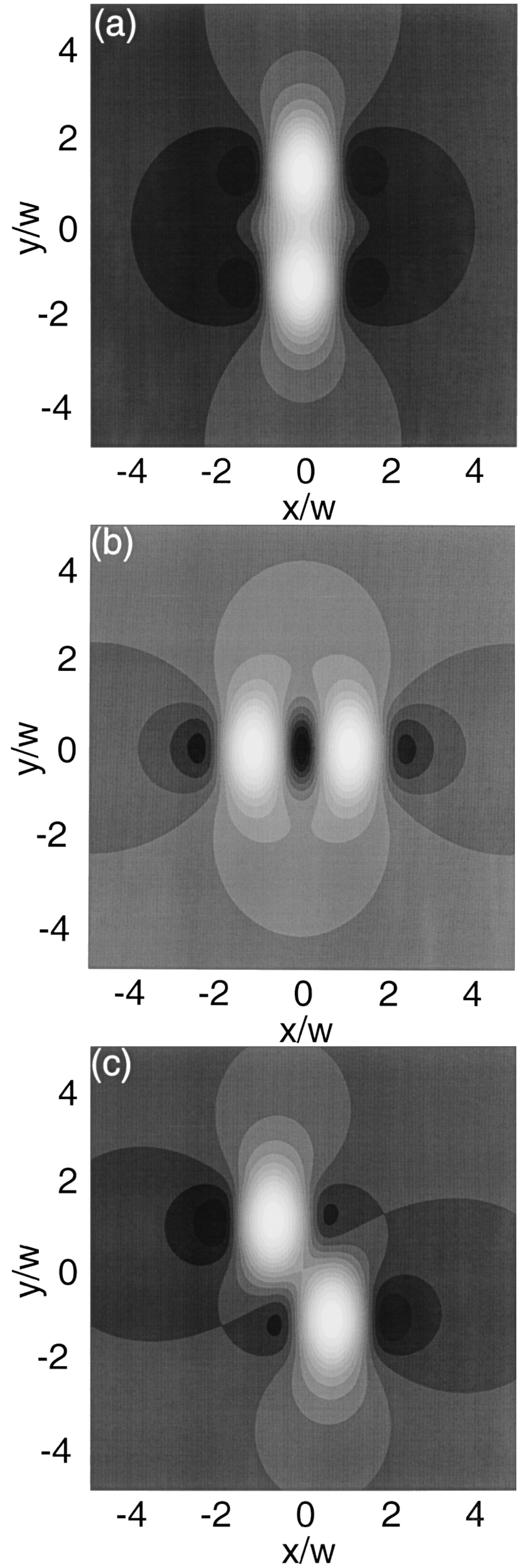

FIG. 1. Transverse distributions of the change in the refractive index $\Delta n/n_0$ for two solitons. Bright regions depict potential wells. The magnitude of the change is $\sim 10^{-4}$. (a) Attracting solitons perpendicular to the external field. (b) Repelling solitons along the external field. (c) Slanted attracting solitons.

the ray can be regarded as the path of a particle with mass n moving in a potential $V = -n$ [18]. The quantization of the Hamiltonian leads to the paraxial wave equation [17], which is the wave counterpart of the paraxial ray equation. The connection with spatial solitons is established by viewing the soliton as a bundle of rays.

In our understanding an optical soliton is a focused beam of light boring an optical path through the crystal. It is composed of many rays, each carrying an appropriate amount of light energy. In the mechanical picture they act as a system of particles. Hence, we assume that the soliton trajectory, as introduced above, represents the trajectory of the center of mass of such a system. The transverse coordinates of the soliton position $\langle x \rangle$, $\langle y \rangle$ act as the canonical coordinates of the solitonic particle. The motion of the particle is governed by the Hamiltonian equations for the center of mass

$$\frac{d\langle x\rangle}{dz} = \frac{1}{n_0}\langle p\rangle, \qquad \frac{d\langle y\rangle}{dz} = \frac{1}{n_0}\langle q\rangle, \tag{11a}$$

$$\frac{d\langle p\rangle}{dz} = \left\langle \frac{\partial n}{\partial x}\right\rangle, \qquad \frac{d\langle q\rangle}{dz} = \left\langle \frac{\partial n}{\partial y}\right\rangle. \tag{11b}$$

Such an interpretation is either approved or disproved by comparison with the full numerical integration of Eqs. (2) and (4). Nevertheless, use of geometrical optics is justified, as the dimension of solitons is large compared to the wavelength, diffraction is absent, and incoherent solitons are considered.

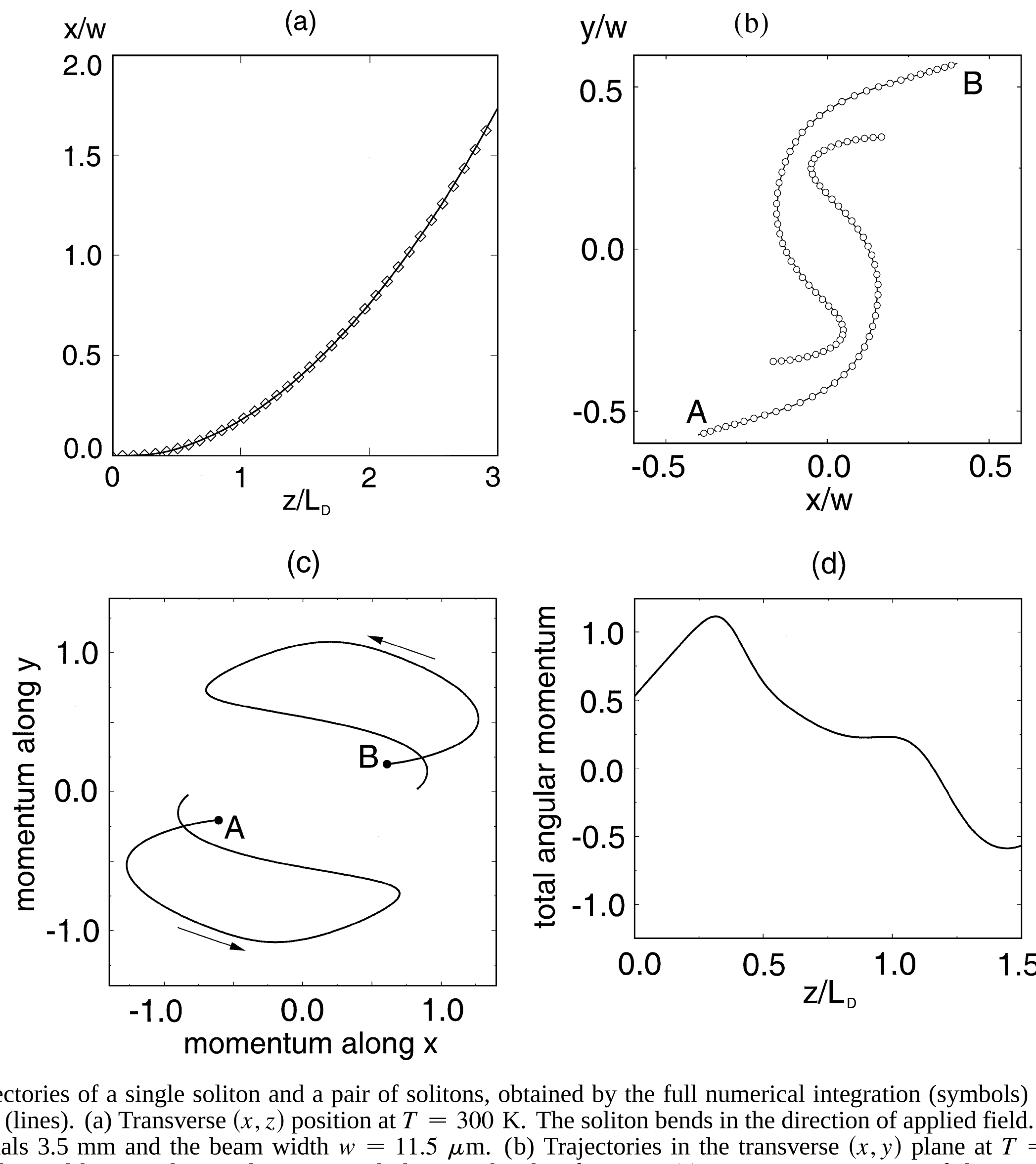

FIG. 2. Trajectories of a single soliton and a pair of solitons, obtained by the full numerical integration (symbols) and the particle approximation (lines). (a) Transverse (x, z) position at $T = 300$ K. The soliton bends in the direction of applied field. The diffraction length L_D equals 3.5 mm and the beam width $w = 11.5$ μm. (b) Trajectories in the transverse (x, y) plane at $T = 0$. The initial positions are denoted by A and B. Solitons rotated about each other for $\sim\pi$. (c) Transverse momenta of the two solitons. They bounced off the potential barrier at the inflection points. (d) Total angular momentum of the pair as a function of the propagation distance. The momentum reverses its sign after each bounce.

Equations (2) and (4) are integrated by the method described elsewhere [16]. The computed value of n is used to evaluate the forces in Eqs. (11), which are then integrated separately. Material parameters used in simulations correspond to the values for SBN crystals found in experiment [9,10]. We launch either one or two Gaussian beams.

When a single beam is launched into the crystal, one first observes self-focusing into a soliton shape, oscillation of the two beam diameters, and bending in the direction of applied field, caused by the temperature-dependent diffusion field in Eq. (4). The launching of two incoherent beams in the plane of applied fields leads to the anomalous interaction of resulting solitons [10]. Overlapping solitons attract and well-separated solitons repel. Beams launched in the plane perpendicular to the direction of applied field only attract [16]. These differences stem from the distribution of the refractive index change, which acts as a potential well in which the pair of solitons self-consistently moves (Fig. 1). Owing to the shape of the potential, the beams launched skewed to the direction of applied field always initially rotate about each other. A question has been raised in the literature [9,10] whether this rotation can be prolonged into the spiraling of solitons. The particle picture provides an answer.

Figure 2 displays beam trajectories of the single and two interacting solitons, computed by the full numerical and the particle methods. Complete agreement is evident. The knowledge of mechanical quantities allows for a simple explanation of many salient features of optical solitons, such as the apparent inertia of intense solitons, attractive as well as repulsive forces between them, and the lack of closed orbits in the transverse plane. Crucial in the picture is the refractive index change, which acts as the potential generating forces between solitons. The total momentum is conserved, but the angular momentum is not, and the solitons perform complicated motion (Fig. 2). They bounce between potential shoulders along the direction of external field, reversing the sense of rotation after each bounce, and eventually are arrested by the potential well perpendicular to this direction. Initial rotation is followed by the oscillation perpendicular to the direction of applied field. Nonconservation of the angular momentum indicates that the transverse trajectories cannot be conical sections. The potential presented in Fig. 1 is anisotropic and the forces acting on solitons are noncentral, preventing closed orbits in the transverse plane. This precludes an indefinite spiraling of solitons, in agreement with our previous results [10,16].

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