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Phase transfer in optical phase conjugation

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Phase variations during a phase-conjugation process are considered in detail, following our own method of analytical solution of the degenerate four-wave mixing including pump depletion and arbitrary phase mismatch between the light interference and the refractive-index gratings. Closed-form expressions are obtained for the phase transfer, and some apparent misconceptions concerning phase changes in optical phase conjugation are discussed.

Great progress in theoretical and experimental studies of optical phase conjugation in photorefractive media has been achieved in the past several years.[1] One of the more important results allowing deeper insight into the nature of nonlinear wave interaction was derivation of exact solutions of the coupled-wave theory of four-wave mixing[2,3] (4WM) including pump depletion and arbitrary phase mismatch between the index and the interference gratings. These solutions, however, describe exactly only intensities of the interacting waves. The information about particular phases and, from the practical point of view, especially the interesting phase of the output phase conjugate wave (PCW), is deeply hidden in these theories or obscured by the use of necessary numerics. In the words of authors of Ref. 2: "The effects of strong nonlinearities on the phases of the output beams are not yet understood, and are the subject of current theoretical efforts."

As a result of these efforts came a paper[4] dealing with the phases of phase conjugation, but in this paper the analysis of the phase transfer was also performed numerically. On the other hand, knowledge about the phase of the PCW is of great practical importance due to applications in the analysis of double-phase conjugate resonators[5] or in the studies of photorefractive properties of electrooptic crystals.[6]

The purpose of this paper is to solve exactly the coupled nonlinear equations governing 4WM in terms of the amplitudes of interacting waves, with particular emphasis on the phase transfer during the process. Consequently, the phases of the waves will be given in closed, analytical form. In addition, using our results, we will clarify some commonly used assumptions regarding the influence of the phases on the conjugation process, and of the intensities on the phase change.

In our studies we apply the same notation and general assumptions about the mixing geometry as in Ref. 2. The coupled equations, describing the transmission 4WM, in the slowly varying envelope approximation and without absorption are of the form

$$A_1' = -\frac{\gamma}{I_0}(A_1 A_4^* + A_2^* A_3) A_4 \ , \qquad (1a)$$

$$A_2^{*\prime} = -\frac{\gamma}{I_0}(A_1 A_4^* + A_2^* A_3) A_3^* \ , \qquad (1b)$$

$$A_3' = \frac{\gamma}{I_0}(A_1 A_4^* + A_2^* A_3) A_2 \ , \qquad (1c)$$

$$A_4^{*\prime} = \frac{\gamma}{I_0}(A_1 A_4^* + A_2^* A_3) A_1^* \ , \qquad (1d)$$

where γ is the wave-coupling constant and I_0 is the total intensity. From Eqs. (1) the following conservation relations result:

$$I_1 + I_4 = d_1 \ , \qquad (2a)$$

$$I_2 + I_3 = d_2 \ , \qquad (2b)$$

$$A_1 A_2 + A_3 A_4 = c \ . \qquad (2c)$$

With the help of these constraints, the coupled equations may be rewritten as follows:

$$A_1' + \gamma A_1 \frac{I_3 + I_4}{I_0} = -\frac{\gamma}{I_0}(A_2^* c - A_1 d_2) \ , \qquad (3a)$$

$$A_2^{*\prime} + \gamma A_2^* \frac{I_3 + I_4}{I_0} = -\frac{\gamma}{I_0}(A_1 c^* - A_2^* d_1) \ , \qquad (3b)$$

$$A_3' + \gamma A_3 \frac{I_3 + I_4}{I_0} = \frac{\gamma}{I_0}(A_4^* c + A_3 d_2) \ , \qquad (3c)$$

$$A_4^{*\prime} + \gamma A_4^* \frac{I_3 + I_4}{I_0} = \frac{\gamma}{I_0}(A_3 c^* + A_4^* d_1) \ . \qquad (3d)$$

Introducing a new set of dependent variables through the transformation

$$A_j \to A_j \exp\left[\gamma \int_0^z \frac{I_3 + I_4}{I_0} dz'\right] \equiv A_j \exp\gamma[X(z)] \ , \qquad (4)$$

the set (3) breaks into two pairs of coupled linear differential equations with constant coefficients. After integration and returning to the original variables, we obtain the following expressions for the amplitudes of the interacting waves:

$$A_1(z)=\frac{A_{10}f_1(d-z)+2cA^*_{2d}f_2(z)\exp\gamma\left[\chi_d-\frac{d}{2}\right]}{f_1(d)}$$

$$\times\exp\gamma\left[\frac{z}{2}-\chi\right], \quad (5a)$$

$$A^*_2(z)=\frac{A^*_{2d}\exp\gamma\left[\chi_d-\frac{d}{2}\right]f_1(z)-2c^*A_{10}f_2(d-z)}{f_1(d)}$$

$$\times\exp\gamma\left[\frac{z}{2}-\chi\right], \quad (5b)$$

$$A_3(z)=\frac{2cA^*_{40}f_2(d-z)}{f_1(-d)}\exp\gamma\left[\frac{z}{2}-\chi\right], \quad (5c)$$

$$A^*_4(z)=\frac{A^*_{40}f_1(z-d)}{f_1(-d)}\exp\gamma\left[\frac{z}{2}-\chi\right], \quad (5d)$$

where $f_1(x)=\Delta\sinh(\mu x)-Q\cosh(\mu x)$, $f_2(x)=\sinh(\mu x)$, $\Delta=d_2-d_1$, $Q=(\Delta^2+4|c|^2)^{1/2}$, $\mu=(\gamma Q/2I_0)$, and A_{10}, A_{2d}, and A_{40} are the given input amplitudes at $z=0$ and $z=d$. To find the explicit form of $\chi(z)$ we can either go back to its definition, Eq. (4), and solve the differential equation for $\chi(z)$, or use the definition for c, Eq. (2c), to obtain

$$\chi(z)=\frac{z}{2}+\frac{1}{2\gamma_r}\ln\left[\frac{F_{12}(z)}{|f_1(d)|^2}+\frac{F_{34}(z)}{|f_1(-d)|^2}\right], \quad (6)$$

where

$$F_{12}(z)=I_{10}\left[f_1(d-z)+\frac{2I_{2d}Q}{D}f_2(z)\right]$$

$$\times\left[\frac{I_{2d}Q}{|c|^2D^*}f^*_1(z)-2f^*_2(d-z)\right], \quad (7)$$

$$F_{34}(z)=2I_{40}f^*_1(z-d)f_2(d-z), \quad (8)$$

with γ_r being the real part of the coupling constant and $D=Q\cosh(\mu d)+I_0\sinh(\mu d)$.

In this manner, we have obtained exact solutions for the amplitudes of the interacting waves expressed in terms of their boundary values and the quantity c. The latter may be found directly from the definition. Substituting expressions (5) into Eq. (2c) and evaluating at the boundary $z=d$, we find c to be a root of the algebraic equation

$$c\,[Q\cosh(\mu d)+I_0\sinh(\mu d)]-A_{10}A_{2d}Q\exp\gamma\left[\frac{d}{2}-\chi_d\right]=0\,. \quad (9)$$

It is worth noting that our solutions (5) together with Eqs. (7) and (9) are equivalent to those previously derived by Cronin-Golomb *et al.* In particular, it is seen immediately that the output phase conjugate reflectivity has exactly the same form as in Ref. 2. The main advantage of the solutions presented here, however, is that they contain exact information of not only the intensities but also of the phases of the waves. The important phase of the output conjugate beam is given by

$$\phi_3(0)=\mathrm{Arg}\left[-2cA^*_{40}\frac{T}{\Delta T+Q}\right], \quad (10)$$

where $T=\tanh(\mu d)$. It is seen that the phase of the PCW depends on the coupling strength γd, the phase of the probe wave, and on the constant c. Note that the same expression for $\phi_3(0)$ also follows from the results in Ref. 4, but there the equation from which the phase of c could be found is missing. Consequently, the phase of the PCW had to be evaluated numerically.

Expression (10) may be used to study the behavior of the phase as a function of all input parameters. We shall demonstrate how our procedure works in the case of 4WM with the $\pi/2$ photorefractive phase shift, where coupling constant γ is real. We see from (5c) that the phase of the PCW is constant along the propagation direction z, and may be expressed in the following form:

$$\phi_3(0)=\mathrm{Arg}(-c)+\mathrm{Arg}(T)-\phi_{40}\,. \quad (11)$$

On the other hand, from Eq. (9) we have

$$\mathrm{Arg}(-c)=\phi_{10}+\phi_{2d}+\pi-\mathrm{Arg}(I_0T+Q)\,. \quad (12)$$

Both of these relations lead to the well-known result, namely that the relative phase $\psi=\phi_4+\phi_3-\phi_2-\phi_1$ of the 4WM process equals 0 or π. However, the question of what do the phases ψ and ϕ_3 depend on turns out not to be trivial. In Ref. 4 the authors claim that "(. . .) if γ is real, $\phi_3(0)=\phi_{10}+\phi_{2d}-\phi_{40}$ and the phase of the conjugate reflection is independent of the intensities of the interacting beams." It is also commonly assumed, and particularly in the analysis of 4WM in terms of intensities and phases, that ψ depends on the sign of the coupling constant, and that it is π for $\gamma>0$ and 0 for $\gamma<0$.[7] Our results make it possible to check these assumptions and to correct them when they are found inappropriate.

From (11) and (12) we have

$$\psi=\mathrm{Arg}T+\pi+\mathrm{Arg}(I_0T+Q)\,. \quad (13)$$

Thus, for positive γ, $T>0$, and, of course, $\psi=\pi$, as assumed. The situation is not so simple when $\gamma<0$, since

$$\psi=\mathrm{Arg}(Q+I_0T)\,, \quad (14)$$

and therefore, the sign of the term on the right-hand side determines the relative phase and the phase of the PCW. It may be shown that when $\gamma d>-2$, this term is always positive and $\psi=0$. However, for large coupling strengths it is possible that this term could become negative, so that $\psi=\pi$, similar to the case of positive γ. This result, substantiated below, shows that the phase of the conjugation reflection may indeed depend on the input intensities of the beams. Moreover, it also shows that the *a priori* assumption that the relative phase only depends on the sign of the coupling constant is also incorrect. Such a claim is probably taken as a generalization of the results of the undepleted pumps approximation. In fact, the small signal theory of 4WM leads to such a conclusion.[3]

Inspection of Eq. (9) also shows that the phase jump from $\psi=0$ to $\psi=\pi$ is closely related to the multistability in phase conjugate reflectivity and that the found coupling-strength value $\gamma d=-2$ actually represents the threshold value for appearance of this effect. For $|\gamma d|<2$, Eq. (9) has only one solution. The relative phase ψ connected with this solution is 0 according to our earlier argument. Above the threshold, i.e., for $\gamma d<-2$, there always exists a range of input intensities such that Eq. (9) exhibits multiple solutions. It can be shown that one of these is referred to the phase $\psi=0$. Two additional ones are connected with the relative phase $\psi=\pi$. This phase change has a profound effect on the interference factor $A_1A_4^*+A_3A_2^*$ in Eqs. (1). For $\psi=0$ these two terms are in phase, and add coherently; for $\psi=\pi$ they are out of phase, interfering destructively. Consequently, instability and multiple solutions appear in the intensities as well.

The occurrence of multiple solutions is depicted in Fig. 1, where the real part of the left-hand side of Eq. (9) is plotted as a function of $|c/I_0|^2$ for different values of the coupling constant. It should be pointed out that the allowed range for $|c/I_0|^2$ is 0–0.25; however, we restricted our attention in Fig. 1 to only the most interesting region where the multiple solutions appear. The vertical dashed line in Fig. 1 is associated with the case $\gamma d=-2.2$, and it separates the regions where $\psi=0$ (to the right) and $\psi=\pi$ (to the left of the line). Thus it is seen that even for the negative (and large) coupling strength that some solutions belong to the $\psi=\pi$ case.

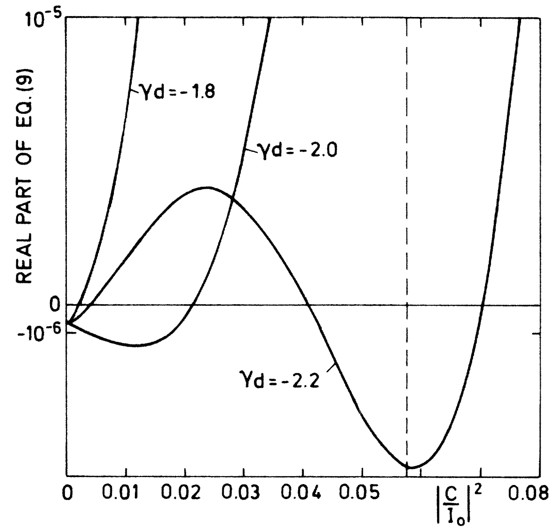

FIG. 1. Illustrating the occurrence of multiple solutions in Eq. (9) for different values of the coupling constant. The modulus of this equation is equivalent to the equation for $|c|^2$ in Cronin-Golomb *et al.* (Ref. 2). When $|\gamma d|<2$, only one solution is possible, leading to the value $\psi=0$ for the relative phase (the displayed case $\gamma d=-2.0$ is chosen slightly below the threshold). However, when $\gamma d=-2.2$, three solutions are observed: two to the left of the dashed divide, corresponding to $\psi=\pi$, and one to the right of the divide, corresponding to $\psi=0$. In all the plots the pump ratio I_{2d}/I_{10} and the probe ratio $I_{40}/(I_{10}+I_{2d})$ are chosen to be 9 and 0.3, respectively, on the logarithmic scale.

In summary, we have studied the problem of the phases in optical phase conjugation in photorefractive media. We have found analytical expressions for the particular phases of the interacting waves. We have also shown that even for a real coupling constant, the phase of the output conjugate wave may depend on the intensities of the interacting beams. Finally, the change in the relative phase is connected with the observed multistability in the phase-conjugate reflectivity.

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