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Beam bending in photorefractive conjugators

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Abstract. – Strong bending of laser beams in photorefractive conjugators is explained using the grating action method and Fermat's principle. A bent bidirectional optical path in the crystal consists of a finite number of kinks formed at the double-phase conjugate interaction regions, and straight segments in between. The device in operation strives to minimize the optical path length and maximize the coupling between beams.

One of the fascinating sights when watching photorefractive (PR) conjugators in action is the strong bending of focused light beams inside the crystal [1–4]. The beams can make tight corners on a single pass through the crystal, approaching 90° turns. To the naked eye they appear to curve continuously, however upon closer inspection it is discovered that the optical paths are piecewise straight, joined by a number of elbows [1–3]. The explanations offered thus far [2, 3, 5] are either incomplete, or rather complicated.

Beam-bending is present in most of PR oscillators, but it is especially spectacular in the bridge conjugator [1]. In fact, the bending is essential for the operation of this device, in that the bridge conjugator does not utilize any other means of bringing the two incoherent input beams together, such as total internal reflections in the crystal or external mirrors outside. There exists only another conjugator similar in this respect to the bridge, and that is the double-phase conjugate mirror (DPCM) [6]. However, the two are different devices in many aspects, the most obvious being that DPCM employs only one four-wave mixing (4WM) interaction region (IR) to achieve the coupling between the incident beams, whereas the bridge employs many. A more subtle, but physically more consequential difference is that the oscillation transition in DPCM is like a second-order phase transition, whereas the one in bridge is like a first-order one.

The purpose of this letter is twofold. On the one hand, we explain the operation of the bridge conjugator, and on the other hand, we establish a paradigm for the mechanism of self-bending of beams observed in other PR conjugators [7–9], as well as in other wave mixing processes [4]. The paradigm is Fermat's principle, adapted to the operation of PR conjugators.

The geometry of interest is presented in fig. 1. It is a symmetric bridge, with the crystal c-axis pointing along the diagonal and a transverse stream of IR. Such a geometry makes

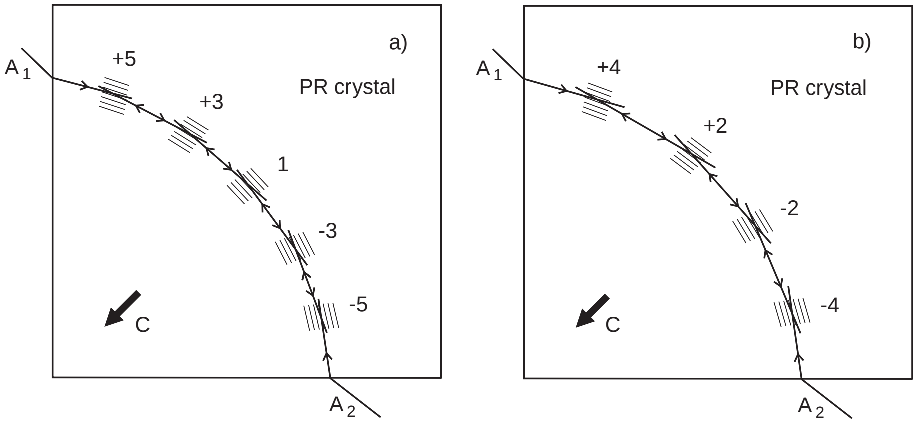

Fig. 1 – Bridge conjugator with (a) 5 interaction regions, (b) 4 interaction regions.

our life easy. The notation is geared to take advantage of the symmetry, since the regions pairwise symmetric across the diagonal are equivalent to each other. The bridges with odd and even number of IR belong to different symmetry classes—the former possess a central region (denoted by 1), whereas the latter have no such region. We assume the envelopes of the mixing beams to be approximated by plane waves, and the wave coupling constant to be real. One can then apply the grating action method [10], according to which all quantities of interest in the wave mixing process are given in terms of one real quantity, the grating action u:

$$u = \frac{1}{d}\int_0^d \frac{\Gamma|Q|}{I}\mathrm{d}z \ . \tag{1}$$

Γ stands for the coupling strength of the process (gain coefficient per unit length times the thickness of IR), $Q = A_1\bar{A}_4 + \bar{A}_2 A_3$ is the amplitude of the grating, A_j are the slowly varying envelopes of the four interacting beams, and I is the total intensity. The bar denotes complex conjugation.

The grating action is intimately tied with Fermat's principle, which requires that the optical path within the device is extremal. The optical path is the line integral of the index of refraction over the propagation path. In a PR crystal the index of refraction contains the contributions from the bulk n_0 plus the change $n_1 \sim |\Gamma Q|/I$, coming from the grating [11]. When integrated along the propagation path, this change gives the (magnitude of) the grating action multiplied by the thickness d of IR. Hence, the physical meaning of the grating action is that it represents the change in the optical path within the crystal, due to the establishment of 4WM gratings.

The grating action u is obtained from the expression [10]

$$(A_{10}\bar{A}_{40} + \bar{A}_{2d}A_{3d} + \text{c.c.})\cot(u) = aI\coth(a\Gamma/2) - I_{10} + I_{2d} - I_{3d} + I_{40} \ , \tag{2}$$

accommodating the four input beams at the 0 and the d side of an IR. a is a constant, to be found from

$$a^2 I^2 = 4|Q|^2 + F^2 \ , \tag{3}$$

where $F = I_1 + I_2 - I_3 - I_4$ is the Poynting flow through the crystal. In the case of devices with one IR, one obtains a system of two algebraic equations. For the devices with more than one IR, say N, one applies the same analysis to each IR (utilizing appropriate boundary

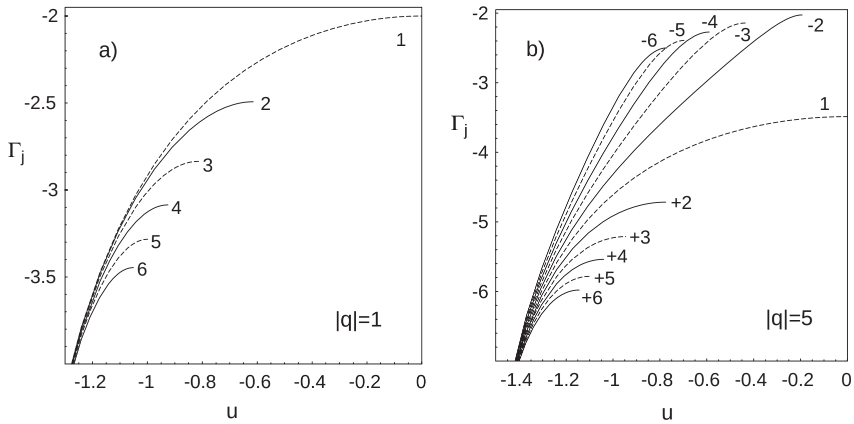

Fig. 2 – (a) Coupling strengths Γ_j as functions of u, for up to 6 regions symmetric ($|q| = 1$) device. (b) Same as (a), but for an asymmetric device, with $|q| = 5$.

conditions), and obtains a system of $2N$ coupled nonlinear algebraic equations. Even though such systems are easier to handle than the original $4N$ wave equations, they are still difficult.

When applied to the bridge with N IR, eqs. (2) and (3) give

$$\cos u_j = \frac{|q|^2 \prod_{i=0}^{j-1} \sin^2 u_i + \prod_{i=j+1}^{N+1} \sin^2 u_i}{2|q| \prod_{i\neq j}^{N+1} |\sin u_i|} \sqrt{1 - a_j^2} \;, \tag{4a}$$

$$1 + a_j \coth\left(\frac{a_j \Gamma_j}{2}\right) = 0 \;, \qquad \text{for} \quad j = 1, ..., N, \tag{4b}$$

where $|q|^2 = I_1/I_2$ is the intensity ratio of incoming beams, and, by definition, $u_0 = u_{N+1} = -\pi/2$. Index j here enumerates the regions. Note that each region has its own coupling Γ_j, which, in general, is different from the others. By the choice of variables, each Γ_j varies from -2 to $-\infty$, and the corresponding u_j varies from 0 to $-\pi/2$. The second N equations are decoupled, and give a_j in terms of Γ_j. Each of the remaining N equations gives u_j in terms of its own Γ_j and all other u_i. The quantities of experimental interest are the transmissivity and the reflectivities at both ends:

$$T = \prod_{i=1}^{N} \sin^2 u_i \;, \qquad R = |q|^2 T \;, \qquad R' = T/|q|^2 \;. \tag{5}$$

We are not aiming to solve these equations, but to establish the threshold and operation conditions of the device. To this end we assume that in the steady state, and for $|q| = 1$, all IR are contributing equally to the working of the device. We call this the equipartition principle, and employ it along with the Fermat principle. It means that all u_j are equal (but Γ_j are unequal). Each of eqs. (4a) then defines a function $\Gamma_j(u)$, some of which are depicted in fig. 2a. The utility of notation introduced in fig. 1 now becomes clear. Looking into eqs. (4a), it is seen that there exist two distinct cases of the bridge conjugator, one with an odd number of IR, and the other with an even number. For any odd N, the equation for the central region is always the same, and equivalent to the equation for the simple DPCM:

$$\cos u_1 = \frac{|q|^2 + 1}{2|q|} \sqrt{1 - a_1^2} \;. \tag{6}$$

Likewise, the equations for the pair of ± 3 regions are always the same:

$$\cos u_{3\sigma} = \frac{|q|^{2\sigma} + \sin^4 u_{3\sigma}}{2|q|^\sigma \sin^2 u_{3\sigma}} \sqrt{1 - a_{3\sigma}^2}\ , \tag{7}$$

where $\sigma = \pm 1$, regardless of the total number of regions. We kept the subscripts on u, to remind us from which pairs they originate, but it should be remembered that all u's are the same. The sequence of regions unwinds in pairs, until N is exhausted. For N even, the sequence starts with the pair of ± 2 regions

$$\cos u_{2\sigma} = \frac{|q|^{2\sigma} + \sin^2 u_{2\sigma}}{2|q|^\sigma |\sin u_{2\sigma}|} \sqrt{1 - a_{2\sigma}^2}\ , \tag{8}$$

and continues in pairs until all the regions are accounted for. Hence, the cases of N odd and N even belong to two symmetry classes of the bridge conjugator (the symmetry operation between the members of any pair is $|q| \to |q|^{-1}$). The classes have nothing in common, however the device can bifurcate from one class to the other. Eliseev *et al.* [2] have described in detail the bifurcation from the $N = 1$ to the $N = 2$ bridge. In our view, the genesis of a bridge from fanning gratings does not proceed by bifurcations from the central region, but by addition of new regions in pairs at both tails of the sequence. Each such addition requires an increasing coupling strength, as the thresholds are increasing, and the process stops after a finite number of steps.

The threshold values for the device are found from the graphs in fig. 2. They are the minimum values of $|\Gamma_j|$ at which the device with $N = j$ regions comes into being. The threshold values, together with the corresponding values of u, and the values of the reflectivity at the threshold, are collected in table I. One notes that the reflectivity is not zero at the threshold for different bridges, with the exception of the $N = 1$ device, which is not a bridge, but DPCM. Hence, the bridges can be turned on only if there are some background fanning gratings, to provide seed. The reflectivity of a bridge suddenly jumps from zero to some finite value at the threshold. This is characteristic of first-order–like phase transitions.

It is worth mentioning that one can also approach the problem by assuming that all Γ_j are the same (then u_j are different). This approach was advocated by Eliseev *et al.* [2]. The two procedures are equivalent for strong $|\Gamma|$ and high $|u|$. The numbers for the equal Γ procedure are also provided in table I. It is seen that the two procedures yield slightly different numbers (except for the starting members of the N-odd and N-even sequences, where they are equal), however the qualitative behavior of the device remains the same.

TABLE I – *Values of the grating action, coupling strength, and the reflectivity at threshold, for the bridge with different number of interaction regions, and for the two solution procedures* (*left side for equal* u, *right side for equal* Γ).

N	u_{th}	Γ_{th}	R_{th}	u_{th}	Γ_{th}	R_{th}
1	−0.0	−2.0	0.0	−0.0	−2.0	0.0
2	−0.615	−2.493	0.111	−0.615	−2.493	0.111
3	−0.815	−2.836	0.148	−0.671	−2.700	0.093
4	−0.925	−3.086	0.166	−0.692	−2.858	0.076
5	−0.998	−3.283	0.176	−0.709	−2.989	0.065
10	−1.172	−3.913	0.195	−0.743	−3.444	0.039
20	−1.291	−4.562	0.205	−0.790	−3.968	0.024

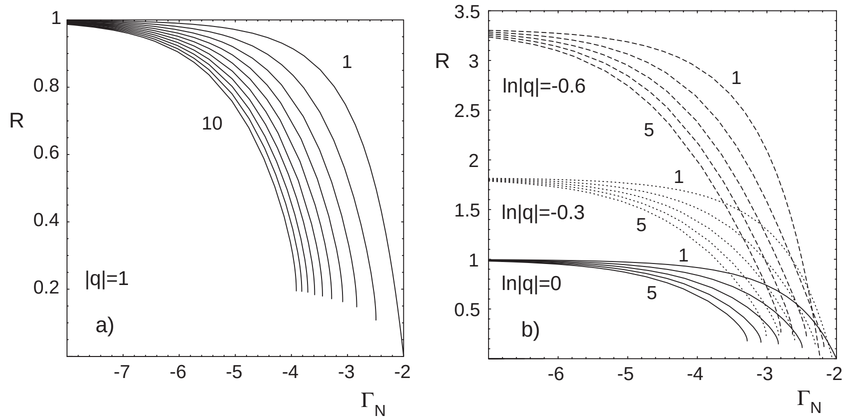

Fig. 3 – (a) Reflectivity as a function of Γ_N for $|q| = 1$. Numbers 1 and 10 denote the number of regions. (b) Same as (a), but for different $|q|$ and for 5 regions.

Figure 3a displays reflectivity as a function of the coupling strength, for different number of regions. Each curve comes into existence after its Γ reaches the threshold value, and we have put Γ_N on the abscissa, to cover the whole physical region of $\Gamma \leq -2$. It is seen that the reflectivity increases as the number of regions increases, but the difference is small for large $|\Gamma|$. The case of unequal input intensities is presented in fig. 3b. It requires a separate discussion.

When $|q| \neq 1$, the symmetry of the device is broken, and the degeneracy of pairs lifted. Figure 2b depicts what happens to the coupling strengths. The pairwise symmetric regions now procure different Γ curves, and possess different thresholds. We distinguish them by the $\pm$ sign, where $+$ is used for the regions upstream, towards the higher intensity pump, and $-$ is used for the regions downstream. It is seen that the regions downstream acquire lower thresholds, and are easier to turn on. Conversely, the regions upstream acquire higher thresholds, and are harder to turn on. Therefore, for a given maximum value of the coupling strength in the crystal, more of the downstream IR will be turned on than the upstream IR. An asymmetric device develops. Asymmetric devices develop also if the direction of the c-axis is tilted, or if an asymmetric launching of the two beams is performed [1]. The reflectivities of both beams for the symmetric device, as functions of the input beam ratio, are shown in fig. 4. They look similar to the experimental curves [1], however the quantitative comparison is difficult to establish, since the experiment is performed for a strongly asymmetric device.

The operation of the symmetric bridge proceeds as follows. After the incoherent beams are launched, a region of overlapping fanning gratings develops, slanted toward the c-axis. Fanning gratings scatter beams they originate from. Among the many scattered replicas of beam 1, amplified will be the ones that eventually scatter in the direction of the phase conjugate to beam 2, and vice versa. Such bidirectional beam pairs are supported by both input beams, whereas other scattered beams are supported by only one input beam. There exist many such bidirectional pairs, as one has to allow for not only the direct scattering (responsible for DPCM), but also for higher orders of multiple scattering (responsible for various bridges). All such pairs compete for the available energy of input beams, and in the end wins the one that utilizes most gain and admits least losses. This is the one which, according to Fermat's

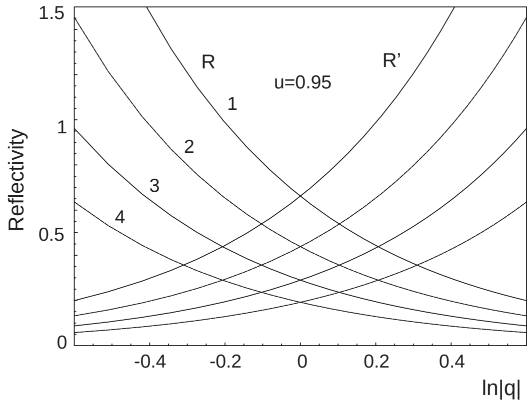

Fig. 4 – Phase conjugate reflectivities at both sides of the crystal, as functions of the beam ratio $|q|$, for the devices with up to 4 interaction regions.

principle, minimizes the total optical path length (OPL):

$$\mathrm{OPL} = \sum_{i=1}^{N+1} n_0 l_i + \sum_{i=1}^{N} |u_i| d_i \ , \tag{9}$$

where the first term is the sum over straight segments, and the second term represents the contribution of the kinks, in the lowest, linear order. l_i are the lengths of straight path segments joining IR, and d_i contain all the factors [11] of dimension length that multiply $|u_i|$. The interplay between the two terms decides which bridge is turned on. The first term carries a lot of weight, but the second term dictates how many kinks there will be in the optical path. For example, for the bifurcation from $N = 1$ to $N = 2$ device [2], the Fermat condition is

$$n_0 l_3 + 2|u_2| d_2 \leq n_0(l_1 + l_2) + |u_1| d_1 \ , \tag{10}$$

where l_1, l_2, l_3 are the sides of the triangle connecting IR, l_3 being the side between the two $|u_2|$ regions. This condition is easily satisfied.

It appears that such a picture would lead to a continuously curved optical path, however this is not the case, as the thresholds increase with the number of regions. The amount of coupling strength within the crystal is always limited. Only those IR will be activated, whose thresholds lie below the level of $|\Gamma|$ available. The device will push towards the higher values of $|u|$, as there the values of $|\Gamma|$ are higher, and consequently the maximum possible number of IR will be activated. Hence, the steady-state operation of the bridge requires that OPL is at the minimum, but at the same time each IR operates at the maximum $|u_j|$ achievable under such conditions. It may happen that a few competing bridges have very close OPL, in which case the device will oscillate in time between the allowed geometries [12].

It would also appear in this picture that there is no reason for the device to depart from the $N = 1$ geometry. The reflectivity attained there is the highest, the threshold is the lowest, and there is no need for a seed. The DPCM threshold is like a second-order phase transition. It is a plausible argument, and were it not allowed, DPCM would have never been observed. However, it is also clear that the bridge is possible (and observed), and to see what happens in reality, one has to account for the details of the geometry and Fermat's principle condition. For example, it may happen that the beams of unequal intensity are launched at such positions and angles, that the angle at their intersection is too large for the formation of DPCM. Then an asymmetric bridge is clearly favored.

Finally, it should be mentioned that a plane-wave theory, such as this one, cannot be expected to offer a complete description of the bending of beams in PR crystals. Self-bending happens in 2WM processes [4], where there are no bidirectional paths, and in the scattering of speckled fields [5], where it is caused by the gradient of the angular spectrum of the beam. A more complete theory should include transverse spatial effects.

Nevertheless, we have established that important features of beam bending in PR conjugators can be explained by a simple grating action theory and Fermat's principle. In particular, we show that the bending of beams in the bridge conjugator results from the establishment of a finite sequence of odd or even number of 4WM interaction regions along the optical path, which minimize the path length and maximize the coupling between the interacting beams.

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