CALCULATION OF TRANSVERSE EFFECTS IN OPTICAL BISTABILITY USING FAST FOURIER TRANSFORM TECHNIQUES

J.V. MOLONEY, M.R. BELIC * and H.M. GIBBS

Optical Sciences Center, University of Arizona, Tucson, Arizona 85721, USA

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The bistable system consists of a unidirectional ring cavity containing off-resonance saturable two-level atoms. Fourier transform techniques permit an efficient solution of the free-space and medium propagation equations. Dynamic outputs showing critical slowing down and overshoot switching explicitly display the role of diffractive coupling in a bistable device.

1. Introduction

Optical bistability is now a well established phenomenon having been experimentally observed in both intrinsic and hybrid devices [1]. Most theoretical models have been confined to a plane-wave analysis [2], often in the mean-field limit. A few authors [3] have included transverse dependencies of the input beam by considering a single mode or a finite mode expansion. Most notable of the latter is the work of Marburger and Felber who consider a single transverse gaussian mode and choose the boundary conditions to match the wavefront curvature. These authors also assume such a high-finesse cavity, that the forward and backward powers in the cavity are taken equal. They conclude that self-focusing can reduce the threshold for bistability. Ballagh et al. [3] also consider a single transverse mode and make the mean-field approximation in order to get analytically tractable results. Their results are in better agreement with the gaussian-beam experiment of Sandle and Gallagher [4] than plane-wave solutions.

The work the closest to our own is that of Rosanov and Semenov [5] which we discovered after our present results. Their model of dispersive optical bistability is a Kerr nonlinearity with no saturable absorption. They do not treat propagation through the nonlinear medium, which they assume to be confined to a single thin sheet. They do employ a one-dimensional fast Fourier transform and treat the very-large-Fresnel case, finding radially dependent switching and avoiding high frequency spatial oscillations by filtering.

In the following, we consider a unidirectional ring cavity configuration with a nonlinear absorbing two level medium. No mean-field approximation is made, and our solution technique rigorously includes all relevant transverse (confined here to one linear dimension x) and longitudinal ring-cavity modes. A saturable homogeneous two-level absorber is assumed but the laser-atom detuning is so large that the optical bistability is primarily dispersive although a finite saturable absorption is always present. Device switch-on (-off) intensities are calculated for two Fresnel numbers and compared with the plane-wave results. Dynamic outputs showing critical slowing down and overshoot switching explicitly display the role of diffraction in a bistable device.

2. Method of solution

We solve the coupled Maxwell-Bloch equations in the limit of fast longitudinal and transverse relaxations of the atomic medium. The resulting wave equation,

* Present Address: Physics Faculty, University of Belgrade, Belgrade, Yugoslavia.

in the paraxial ray approximation, describing propagation though the nonlinear medium can be written [6]

$$\left[\frac{\partial}{\partial \zeta} - \frac{\mathrm{i}(\ln 2)\nabla_t^2}{4\pi F}\right]\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta) = -\tfrac{1}{2}\alpha \mathcal{L} m(I(\boldsymbol{\rho}, \zeta))\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta), \tag{1}$$

where the source (polarization) term on the RHS is derived from the steady-state Bloch equations. The scaled variables $\boldsymbol{\rho}$ and ζ are defined in terms of the original x, y, z cartesian coordinates as follows: $\boldsymbol{\rho} = \hat{x}x/w_0 + \hat{y}y/w_0$; $\zeta = z/\mathcal{L}$. The transverse laplacian is $\nabla_t^2 = w_0^2(\partial^2/\partial x^2 + \partial^2/\partial y^2)$.

$\mathcal{L}$ is the total path length, w_0 is the minimum beam waist at the input mirror, and the Fresnel number is defined as $F = w_0^2(\ln 2)/\lambda\mathcal{L}$. The off-resonance "effective" absorption per pass, α, is defined as

$$\alpha = \alpha_0/(1 + \Delta^2), \quad \Delta = (\omega - \omega_{ab})/\gamma_\perp,$$

and α_0 is the on-resonance field absorption coefficient.

$$m(I(\boldsymbol{\rho}, \zeta)) = (1 + \mathrm{i}\Delta)\left[1 - \frac{I(\boldsymbol{\rho}, \zeta)}{1 + \Delta^2 + I(\boldsymbol{\rho}, \zeta)}\right]$$

is the nonlinear saturable term including absorption and refraction. A systematic derivation of eq. (1), showing it to be accurate to lowest order in $\lambda/(2\pi n_0 w_0)$ is given in refs. [6–8]; these authors conclude that eq. (1) should be an excellent approximation for all problems at optical frequencies.

We note in passing that our problem with the ring cavity boundary conditions applied, corresponds precisely to the limit studied recently by Ikeda [9] in the plane-wave approximation. Indeed, we have observed additional instabilities (bifurcation sequences) on the normally stable branches in appropriate limits, and these will be the subject of a separate publication [10]. In the present article we analyze conventional bistable behavior by choosing our parameters appropriately.

Our solution of eq. (1) follows in the spirit of recent work on unstable resonators (amplifiers) in refs. [6–8] and [11]. We now summarize briefly the main points. The field propagation in the ring cavity can be divided into (a) free-space propagation in which diffraction is described by the solution of the homogeneous wave equation

$$\left[\frac{\partial}{\partial \zeta} - \frac{\mathrm{i}(\ln 2)\nabla_t^2}{4\pi F}\right]\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta) = 0, \tag{2}$$

and (b) nonlinear medium propagation requiring the solution of eq. (1). Solving eq. (2) using Fourier transforms allows us to write a free-space propagator (FSP)

$$\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta) = (\mathrm{FT})^{-1}\exp(-\tfrac{1}{2}\mathrm{i}q^2\zeta)(\mathrm{FT})\boldsymbol{\xi}(\boldsymbol{\rho}, 0), \tag{3}$$

where (FT)[$(\mathrm{FT})^{-1}$] refers to the two dimensional Fourier transform [inverse]. Eq. (3) can be rapidly solved for an arbitrary input field, $\boldsymbol{\xi}_{\mathrm{I}}(\boldsymbol{\rho}, 0)$ by using a fast Fourier transform code (FFT).

Returning to eq. (1) for the nonlinear medium propagation, we formally integrate it to yield

$$\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta) = T_\zeta \exp\left[\mathrm{i}\zeta\nabla_\tau^2 - \frac{\alpha\mathcal{L}}{2}\int_0^\zeta m(I(\boldsymbol{\rho}, \zeta'))\mathrm{d}\zeta'\right] \times \boldsymbol{\xi}(\boldsymbol{\rho}, 0) \tag{4}$$

where T_ζ is a ζ ordering operator and $\nabla_\tau^2 = [(\ln 2)/4\pi F]\,\nabla_t^2$. The propagator in eq. (4) can be solved to third-order accuracy in ζ by rewriting it as [6]

$$\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta) = \exp(\tfrac{1}{2}\mathrm{i}\zeta\nabla_\tau^2) \times \exp\left[-\frac{\alpha\mathcal{L}}{2}\int_0^\zeta m(I(\boldsymbol{\rho}, \zeta'))\,\mathrm{d}\zeta'\right] \times \exp(\tfrac{1}{2}\mathrm{i}\zeta\nabla_\tau^2)\boldsymbol{\xi}(\boldsymbol{\rho}, 0). \tag{5}$$

In practice, we split our medium up into "absorber sheets" of length $\Delta\zeta$ and solve the "difference" scheme

$$\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta_{n+1}) = \exp(\tfrac{1}{2}\mathrm{i}\Delta\zeta\nabla_\tau^2) \times \exp\left[-\frac{\alpha\mathcal{L}}{2}\int_{\zeta_n}^{\zeta_{n+1}} m(I(\boldsymbol{\rho}, \zeta'))\,\mathrm{d}\zeta'\right] \times \exp(\tfrac{1}{2}\mathrm{i}\Delta\zeta\,\nabla_\tau^2)\boldsymbol{\xi}(\boldsymbol{\rho}, \zeta_n). \tag{6}$$

Eq. (6), which is our nonlinear medium propagator, operatively entails the following three steps: (a) free-space propagate $\Delta\zeta/2$ into the absorber sheet, (b) calculate the nonlinear medium contribution over the $\Delta\zeta$ interval, and (c) free-space propagate the remaining $\Delta\zeta/2$ up to the next absorber sheet boundary. In summary, we calculate the initial internal field $\xi(\boldsymbol{\rho}, 0)$ at the input mirror $(\equiv\sqrt{T}\xi_I(\boldsymbol{\rho}, 0))$, propagate it around the cavity using eqs. (3) and (6) taking into account the boundary conditions at the mirrors, return it to the input mirror, increment it by $\sqrt{T}\xi_I(\boldsymbol{\rho}, 0)$, and repeat until the transmitted field $\xi_T(\boldsymbol{\rho})$ has reached a steady state. In this manner we can study the dynamic approach to the steady state.

A major advantage of the computational scheme outlined above is that all relevant modes, transverse and longitudinal, are rigorously included in the treatment. No mean-field approximation is invoked (indeed we observe that propagation effects can be significant) and arbitrary cavity geometries (e.g. curved mirrors) can be considered. The corresponding plane-wave result is obtained by dropping the laplacian terms in eq. (6) and using the "plane-wave propagator"

$$\xi(\zeta_{n+1}) = \exp\left[-\frac{\alpha\mathcal{L}}{2}\int_{\zeta_n}^{\zeta_{n+1}} m(I(\boldsymbol{\rho}, \zeta'))\, d\zeta'\right] \xi(\zeta_n). \quad (7)$$

Finally, we note that the computational scheme adopted in refs. [6–8] and appropriately modified here for optical bistability represents a significant saving in computational effort over conventional numerical difference schemes. The use of the FFT scheme means that computational time goes as $N \log N$ instead of N^2 for conventional approaches (N represents the number of grid points).

3. Results and discussion

Due to memory and speed limitations of our computer, we have confined our preliminary calculation to a single transverse cartesian dimension, i.e., $\nabla_t^2 = w_0^2\partial^2/\partial x^2$ only. We see no reason to believe that inclusion of the full two-dimensional diffraction term will lead to substantial differences in the results reported here. Table 1 summarizes our results for the plane-wave versus gaussian profile ring bistable device. The parameters of the calculation are given in the caption. The main point to note from the table is that a significant intensity increase is required to switch the system to the upper branch as the Fresnel number is decreased. The percent transmission and

Table 1

Input profile	F	$I_\uparrow$	$I_\downarrow$	%Trans
Plane-wave	∞	21.8	5.5	16%
Gaussian	0.55	36.2	10.0	33%
Gaussian	0.055	122.2	40.0	6%

Gaussian values are beam-center values.
Parameters used in calculation: $\alpha_0 = 100$, $\Delta = 5$, $L_{NL}/\mathcal{L} = 0.15$, where L_{NL} is the length of the nonlinear medium. The laser cavity detuning ϕ is 0.4 ($0.2/\pi$ times the free spectral range). The intensity reflection coefficient R of both input and output mirrors is 0.9.

$I_\uparrow$ and $I_\downarrow$ quoted in the table are for the on-axis intensity for the gaussian case. The increase in switch-on intensity $I_\uparrow$ with decreasing Fresnel number is intuitively obvious: increasing diffraction in the gaussian beam leads to less efficient feedback in the device. Another important point to note is that the system switches on simultaneously out to very large radii in the transverse dimension (see fig. 1).

Simultaneous switch-on out to a large radius was recently observed [12] in a GaAs Fabry-Perot etalon with $F \approx 24$ and in the limit of a medium relaxation slow compared with the cavity response, i.e., the reverse of the present limit.

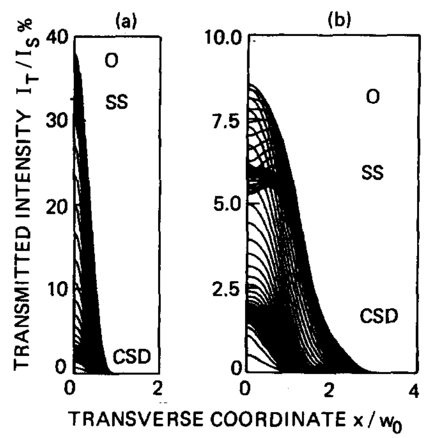

Fig. 1. Transverse profiles, one each round trip, showing critical slowing down (CSD), overshoot switching (O), and steady-state (SS) under the same conditions as table 1. (a) $F = 0.6$, $I_I(x = 0) = 36.2\, I_s$, and (b) $F = 0.06$, $I_I(x = 0) = 122.2$. Figs. 2 and 3 show the corresponding time dependences for the on-axis intensity.

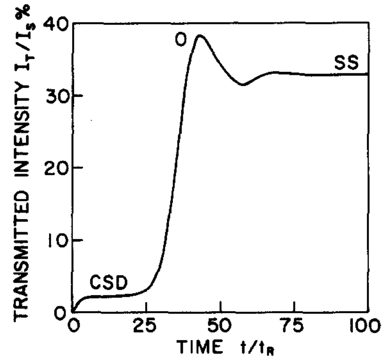

Fig. 2. Dynamics of on-axis intensity showing critical slowing down and overshoot switching for $F = 0.6$ and $I_I(x = 0) = 36.2$.

Figs. 1(a) and (b) show the dynamical switching of the system to the high transmission branch for the cases $F = 0.06$ and 0.6, respectively. Each transverse output in these figures represents the transmitted intensity profile at the output mirror after a single cavity round trip. The input intensity I_I is in the neighborhood of the critical point for switch-on and these dynamic outputs display critical slowing down and overshoot switching. Figs. 2 and 3 represent dynamic outputs of the on-axis intensities of figs. 1(a) and (b) respectively, in units of the cavity round-trip time t_R.

In summary, dispersive optical bistability in a ring cavity has been studied in the limit of very fast medium response and including a transverse (x) dimension. The principal result is that strong diffractive coupling does not destroy bistability, in fact it results in simultaneous switching out to large radius. The

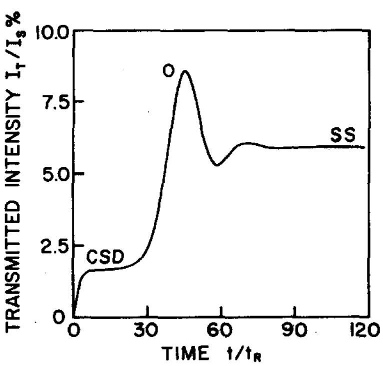

Fig. 3. Dynamics of on-axis intensity showing critical slowing down and overshoot switching for $F = 0.06$ and $I_I(x = 0) = 122.2$.

present one-transverse-dimension Fourier transform method has recently been extended to the large effective Fresnel number case and the switching is found to be radially dependent [13].

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