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Oscillation versus amplification in double phase conjugation

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Abstract

The question whether double phase conjugator is an oscillator or an amplifier is addressed by a careful numerical investigation of the operation of the device in the regime of diminishing seeds. It is established that when the transverse and dynamical effects are included, the double phase conjugator can be both an oscillator and an amplifier, depending on the values of the diffraction and the convection parameters. Dynamics at threshold is investigated by two independent numerical methods, to find that competitive modes other than phase conjugate can grow and be stabilized.

Keywords: Optical phase conjugation; Photorefractive oscillators

A heated controversy arose recently [1–4]: Is the double phase conjugate mirror (DPCM) an oscillator or an amplifier? The difference between the two amounts to whether a finite phase conjugate (PC) output can be obtained from zero input, or a finite input seed is always needed. We set out here to investigate this question.

In general, an oscillation grows out of noise once the device's coupling strength is above a well defined threshold. Finite PC output is obtained in the limit of no seed input. In amplification there is no such threshold and any finite seed is amplified to some finite output. Here no seed means no output. We investigate the operation of DPCM as the seeds are decreasing and the coupling strength is increasing. We show that depending on the relevant parameters describing the process, DPCM can be both an oscillator and an amplifier. We also display that at very high couplings and very low seeds (at the level of fanning instabilities) non-PC conical modes can appear and be stabilized.

In the plane-wave (PW) case it is agreed that DPCM is an oscillator [5]. The controversy arose when transverse analyses of DPCM were attempted. The initial analysis of Zozulya et al. [1] indicates that the transverse DPCM is a convective amplifier. A more recent and a more complete analysis by the same group, both experimental and numerical [2], reaffirms that conclusion. Another group [3], considering a transverse model of their own, and backed by their own experimental evidence, claims that DPCM is an oscillator. The theory presented in Ref. [4], uses the vectorial nature of coupled wave equations to show that DPCM is an oscillator. Thus, conflicting analyses have been performed and polarized points

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of view have been reached. The situation seems confusing and a fresh attempt seems warranted.

Our analysis starts at the scaled paraxial wave equations describing DPC process with a minimum of relevant variables and parameters [6]:

$$\partial_z A_1 + \beta \partial_x A_1 + i\phi \partial_x^2 A_1 = Q A_4 , \quad (1)$$

$$\partial_z A_2 + \beta \partial_x A_2 - i\phi \partial_x^2 A_2 = \bar{Q} A_3 , \quad (2)$$

$$\partial_z A_3 - \beta \partial_x A_3 - i\phi \partial_x^2 A_3 = -Q A_2 , \quad (3)$$

$$\partial_z A_4 - \beta \partial_x A_4 + i\phi \partial_x^2 A_4 = -\bar{Q} A_1 , \quad (4)$$

where A_2 and A_4 are the input beams, A_1 and A_3 are the corresponding PC signals. β is the relative transverse displacement that accounts for the convective effects in DPC. It is defined [6] as $\beta = \theta d/\omega_0$, where θ is the half angle at the beams intersection, d is the crystal thickness, and ω_0 is the beams spot size. ϕ is the parameter controlling diffraction, $\phi = (4\pi F)^{-1}$, where $F = \omega_0^2/\lambda d$ is the Fresnel number, and λ is the wavelength in the medium. z is the paraxial axis and x is the transverse coordinate. We restrict our analysis to one transverse dimension, with minimal loss of generality. The bar denotes complex conjugation and Q is the amplitude of the grating that is generated in the crystal. We should mention that at least part of the oscillation versus amplification controversy stems from the fact most of the initial accounts took only convective or only diffractive effects into consideration. We believe that a complete picture should include both of these effects.

The temporal evolution of Q is approximated by a relaxation equation of the form:

$$\tau \partial_t Q + Q = \frac{\Gamma}{I}\left(A_1 \bar{A}_4 + \bar{A}_2 A_3 \right) , \quad (5)$$

where τ is the relaxation time of the grating, I is the total intensity, and Γ is the PR coupling strength (coupling constant times the crystal thickness). Both Γ and β can be positive or negative, however we restrict our attention only to the positive values. In writing Eq. (5) it is assumed that the relative transverse derivative $\partial_x Q/Q$ of the grating amplitude is small as compared to the product of the Debye screening wavenumber and the transverse spot size of any of the mixing beams. In other words, we assume that the characteristic length over which the grating amplitude changes in the transverse direction is large as compared to the grating spacing.

The boundary conditions are that the four initial amplitudes C_{1-4} are launched into the crystal. The transverse amplitude profiles are assumed to be displaced Gaussians, with parameters that take into account noncollinear propagation of the beams:

$$A_{1,4}(x,0) = C_{1,4} G(-\zeta, x \mp \beta/2) , \quad (6)$$

$$A_{2,3}(x,d) = C_{2,3} G(\zeta, x \pm \beta/2) , \quad (7)$$

where $z = 0$ and $z = d = 1$ denote the entry and the exit face of the crystal and $G(\zeta,\rho)$ is the Gaussian beam function [6]. Here ζ represents the beam curvature parameter. In DPC the initial PC beams C_1 and C_3 are not supplied externally, they arise from the noise in the crystal. Therefore, we seed the values of C_1 and C_3 and monitor their influence on the process. We assume that $|C_1|^2 = |C_3|^2 = \epsilon$ and decrease gradually the values of ϵ (up to 10^{-9}). We tried other seeding strategies (random noise generation) and obtained the same results at the point of vanishing seeds.

Analytical treatment of Eqs. (1)–(4) and (5) is not possible. Different numerical procedures are invented [2,3,6]. We employ two completely independent numerical methods, to control numerical instabilities that might arise owing to dynamical instabilities. Such a procedure is recommended in view of the possibility that instabilities seen in a numerical simulation may come from the numerical method employed, and not from the system under investigation [7]. One of the methods is a beam propagation method [6] based on fast Fourier transform (FFT), the other is a Crank-Nicholson (CN) procedure. Both agree excellently in steady state cases and display qualitatively similar results when instabilities are encountered.

Steady state PW case can be solved analytically [8]. Moreover, it can be compared to a second order phase transition [9]. The exit PC fields at the $z = 0$ and the $z = 1$ faces of the crystal are given by [8]:

$$A_{30} = C_2 \sin(u) , \quad A_{1d} = C_4 \sin(u) , \quad (8)$$

where u is the total grating action:

$$u = \int_0^1 \frac{\Gamma |Q|}{I} \mathrm{d}z , \quad (9)$$

which is also connected with the order parameter a.

For symmetric boundary conditions this connection is very simple:

$$\tan(u) = \sinh\left(\frac{a\Gamma}{2}\right). \qquad (10)$$

The order parameter a is found from the transcendental equation:

$$a = \tanh\left(\frac{a\Gamma}{2}\right), \qquad (11)$$

and a sharp threshold condition on the coupling strength $\Gamma_{th} = 2$ follows from this equation. The quantity becoming ordered at the threshold point is the grating amplitude Q:

$$2|Q| = a\left(|C_2|^2 + |C_4|^2\right)\sin(2\theta), \qquad (12)$$

where $\tan(\theta) = \exp(a\Gamma z - a\Gamma/2)$. The beam seeds here play the role analogous to the role of external magnetic field in the ferromagnetic phase transition. The analogy with phase transitions is rather formal and it does not hold in the general transverse case. We use these results as a check on our numerics.

In numerical simulations one always needs a finite seed to start up the process, however one can obtain useful information from the way in which the system behaves as the seed is getting smaller. Based on such an information and confirmed by theoretical results, we conclude that in the PW approximation DPCM is an oscillator with a sharply defined gain threshold. Seeds are only needed as an initial push. If they are turned off after PC beams are obtained, the reflectivity remains high. Reflectivity levels attained do not depend on the seed, with marked saturation owing to depletion of pumps. However, below the threshold the reflectivity depends directly on the seed, going to zero as the seed is diminished.

In the transverse case with $\phi \neq 0$ and $\beta = 0$, DPCM is a diffractive oscillator. The gain threshold is not well defined anymore. Nonetheless, the existence of an oscillation threshold for each value of the

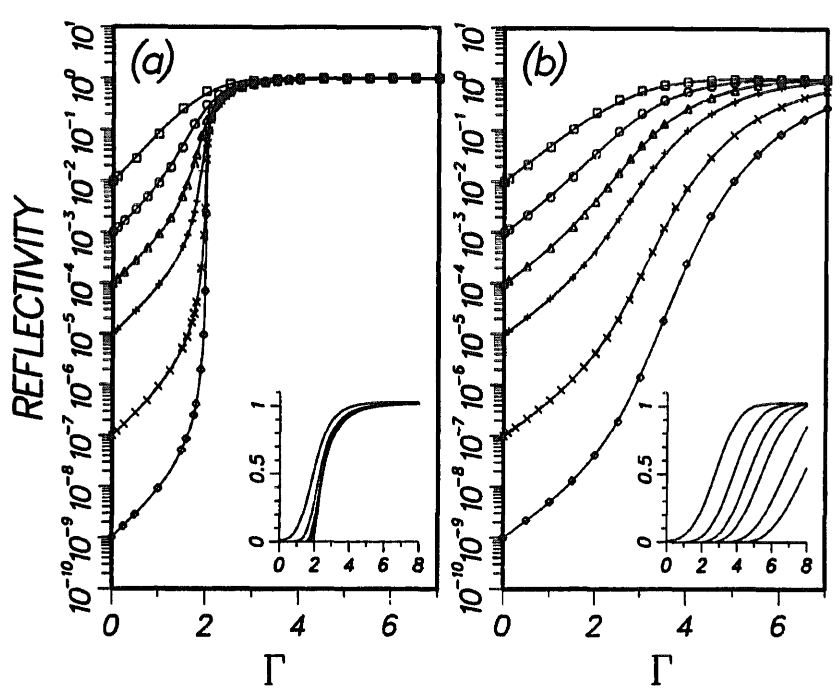

Fig. 1. Total reflectivity versus coupling strength on the logarithmic scale, for different values of the seeding, and for two values of the transverse displacement: (a) $\beta = 0.05$, (b) $\beta = 1$. The value of the diffraction parameter is fixed at $\phi = 2.72 \times 10^{-4}$. The amplitudes of the Gaussian pump beams are chosen equal, $C_2 = C_4 = 1$, while the input of PC beams $|C_1|^2 = |C_3|^2 = \epsilon$ is varied. The value of ϵ for each curve can be inferred from the intercept of the curve with the REFLECTIVITY axis at $\Gamma = 0$. The points indicated are the calculated values, whereas the curves are polynomial fits drawn to guide the eye. The insets depict the same information on the linear scale. While in (a) the existence of a gain threshold at $\Gamma = 2$ is evident, in (b) there is no such threshold, and the amplification of each seed proceeds smoothly.

diffraction parameter ϕ is evident [6]. Further increase in ϕ leads to complicated transverse beam profiles and the device ceases to be a PC mirror. It acts as an amplifier, albeit a poor one. We term such a device a diffractive amplifier. In general, $\beta = 0$ does not mean that the beams are actually collinear. It means that the ratio of the angle at the beams intersection to the angular spread of the beams is small, and hence neglected. The influence of the convective term then can be neglected as compared to the influence of the diffractive term. Such a situation is easily obtained in experiments with thin crystals and wide beams.

When the influence of the transverse displacement is included, then for low values of β the device remains a convective oscillator up to a critical value β_c. Above the critical value of the transverse displacement the device becomes a convective amplifier. Similar to the gain threshold, the critical value of β broadens into a critical region, and the transition from an oscillator behavior to an amplifier behavior is more gradual. This is visible in Fig. 1, which depicts the growth of different beam-seeds for the same value of $\phi = 2.72 \times 10^{-4}$ and for two values of β, one above the critical value and one below the critical value.

An independent test for the oscillation versus amplification is to cut off the seed once the steady state is reached. If the reflectivity attained drops to zero, then the device is an amplifier, if the reflectiv-

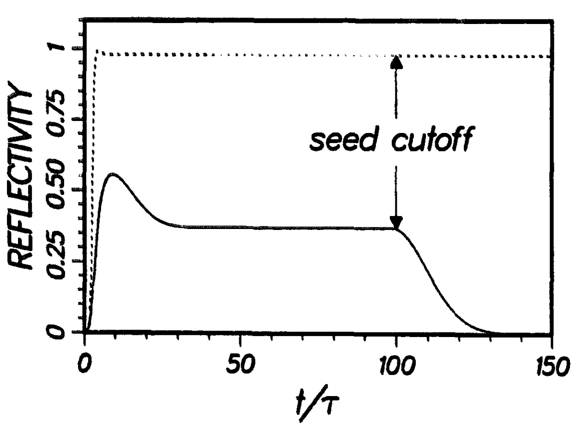

Fig. 2. Reflectivity as a function of time for two values of β. At $t = 100\tau$ the PC seed is cut off (suddenly changed from $\epsilon = 10^{-5}$ to $\epsilon = 10^{-12}$). For $\beta = 0.05$ (dashed line) the reflectivity is unchanged whereas for $\beta = 1$ (solid line) the reflectivity drops to zero. Here $\Gamma = 5$.

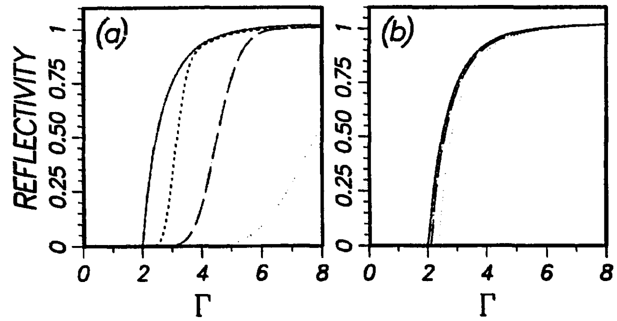

Fig. 3. Reflectivity as a function of the coupling strength, for different values of the transverse displacement β (solid line: $\beta = 0.05$, dashed: $\beta = 0.25$, chain-dotted: $\beta = 0.5$, dotted: $\beta = 1$). (a) Double phase conjugate mode, for $\phi = 2.72 \times 10^{-4}$. (b) Conical mode, for $\phi = 0.01$. The value of the seed is fixed at $\epsilon = 10^{-9}$. The same plots are obtained either by the FFT or the CN method. Other parameters as in Fig. 1.

ity remains high, it is an oscillator. Such a test is performed in Fig. 2, which displays the reflectivity of DPCM before and after the seed is cut off for the two mentioned values of β.

Fig. 3 presents a change in the integrated reflectivity as β is varied, for fixed ϕ, and a fixed seed ($\epsilon = 10^{-9}$). Apart from corroborating the change in the nature of PC process, this figure displays the existence of stable conical modes. The reflectivity of one such non-PC mode that can also be supported by the system is depicted in Fig. 3(b). This competing mode, arising from the noise, is an oscillation. When a conical mode is oscillating, the device is not a PC mirror and instabilities can occur. We discuss such instabilities and mode competition later.

The convective flow of energy out of the interaction region helps resolve the controversy between oscillation and amplification. It represents a mechanism for inhibition of oscillations [1]. In the PW case such a mechanism is absent. Another important mechanism is the multimode operation of DPCM when transverse dimensions are accounted for. Different spatial modes have different oscillation thresholds. When more than one mode can oscillate, the oscillation does not start at a particular value of coupling, but is turned on gradually over an interval. The transverse model changes the sharp transition at the threshold into a more gradual continuous transition.

As the threshold region is approached, critical slowing down is observed. For small ϵ it takes very long times to achieve convergence. As β and ϕ increase, DPCM changes from oscillator-like behavior to amplifier-like behavior, and for high values of the parameters the device might not be a DPCM at all. Thus, having to choose between convective amplifier and optical oscillator in describing DPCM, we believe that the appropriate choice is convective oscillator.

Our results concerning oscillation versus amplification in DPCM can be summarized as follows (Fig. 4). For $\phi = 0$ and $\beta = 0$ DPCM is an oscillator. For $\phi \neq 0$ and $\beta = 0$ it is a diffractive oscillator. For high values of ϕ (of the order of 1) the device is not a PC mirror. It can best be described as a diffractive amplifier. For $\phi = 0$ and $\beta \neq 0$ it is a convective oscillator up to a critical transverse displacement β_c (again of the order of 1). Above β_c DPCM is a convective amplifier. For $\phi \neq 0$ and $\beta \neq 0$ the situation is not so clear. Both diffractive and convective effects contribute to the behavior of the device. For each Γ above 2 there seems to exist a critical curve or a critical region in the (ϕ,β) plane below which the device acts as an oscillator (convective and/or diffractive), and above which it acts as an amplifier (convective and/or diffractive). Owing to critical slowing down, the investigation of such a critical curve is computationally expensive.

For weak seeds and strong couplings, complicated spatio-temporal phenomena occur. Independent of the numerical method applied, instabilities tend to set in, with fanned unstable outputs (Fig. 5). The reflectivity does not settle onto any specific value, but

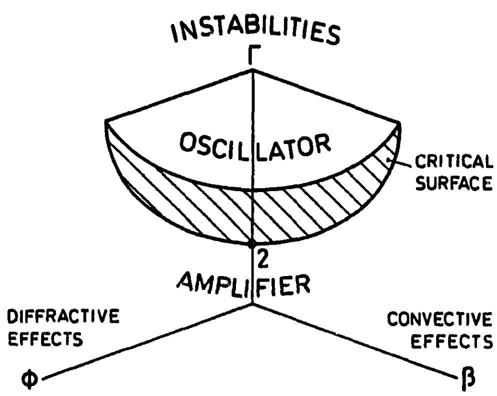

Fig. 4. Displaying qualitatively the behavior of DPCM in the (ϕ,β,Γ) parameter space. For high values of Γ instabilities can occur.

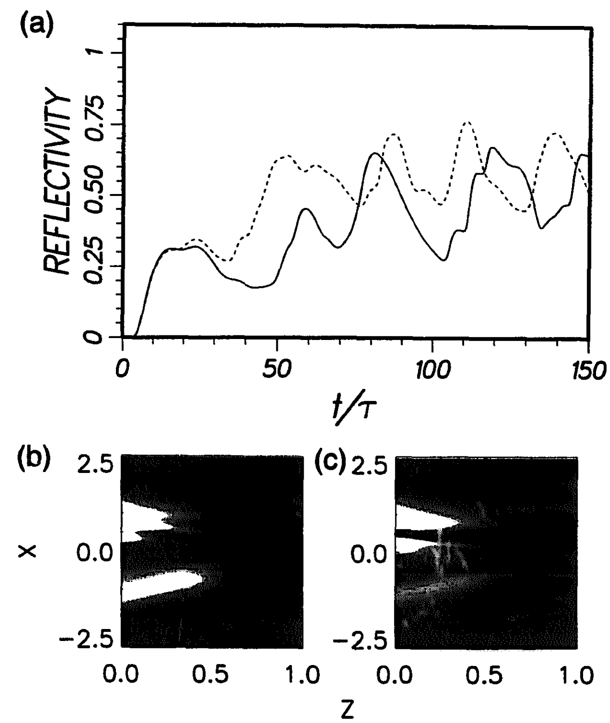

Fig. 5. (a) Dynamics of the total reflectivity, obtained by two independent numerical methods. Solid line: FFT method, dashed line: CN method. The parameters for both simulations are: $\Gamma = 6.5$, $\beta = 1$, $\phi = 0.001$, $\epsilon = 10^{-9}$. (b) Spatial distribution of the PC beam I_3 in the crystal at time 81τ, obtained by the FFT method. (c) Spatial distribution of I_3 obtained by the CN method after 87τ periods. It is seen that qualitatively both methods offer similar behavior, with similar transverse profiles, however the dynamics of the two methods proceeds at different paces.

wanders around in time. Many spatial modes, allowed by the geometry of the process, compete for the energy of the pumps, and an irregular oscillation ensues. We stress the fact that this irregular behavior is observed for real Γ. Previously such irregularities were observed only when the coupling strength is made complex [6].

With a slight change in a parameter (in this case ϕ), a mode other than PC can win the competition, and grow out of the cone of emissions allowed by the DPC geometry. In our one-dimensional transverse case, the cone is along the directions of the two pumps. Generally, the cone collapses in the direction of the maximal gain, which is close to the direction of pumps. In one case a PC beam is obtained, in the other a stable conical oscillation [10]. This is displayed in Fig. 6. The conical emission is an oscilla-

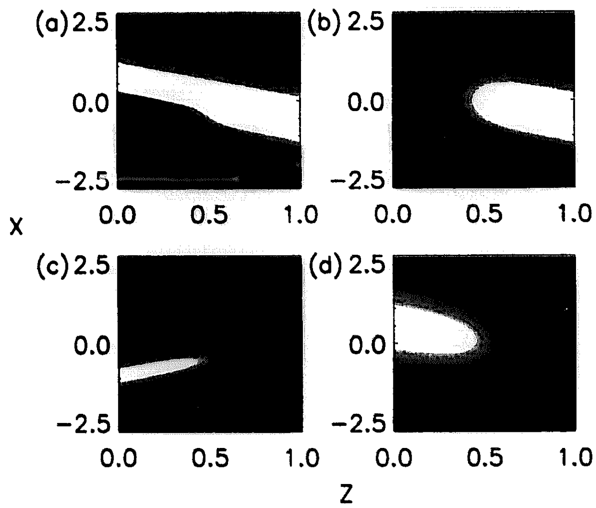

Fig. 6. Stable transverse spatial profiles of (a) I_2 and (b) I_3, for $\phi = 2.72 \times 10^{-4}$ (the PC mode), and (c) I_2 and (d) I_3, for $\phi = 0.01$ (the conical mode). The PC and conical modes are not exactly in the directions of pumps, but are slightly shifted. Identical outputs are obtained, either by the FFT method or by the CN method. The other parameters are $\Gamma = 5$, $\beta = 1$, $\epsilon = 10^{-5}$.

tion, with a clearly visible threshold (Fig. 3b). This threshold looks very similar to the threshold of a PC mode, except that it persists for high β. The conical mode drains energy from the A_2 pump more efficiently than the competing PC mode and is consistent with the available theoretical and experimental evidence [10].

We also find that the inclusion of finite lateral extension of beams in a PC process lowers the reflectivities, causes the appearance of traveling transverse waves and defect-mediated turbulence, improves the understanding of frequency shifts in PR oscillators, and accounts for experimentally observed asymmetry between the transmissivities of the crystal along the beams incidence. These findings are reported elsewhere [11,12].

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