Dynamic counterpropagating vector solitons in saturable self-focusing media

K. Motzek,[1] Ph. Jander,[2] A. Desyatnikov,[2] M. Belić,[3] C. Denz,[2] and F. Kaiser[1]
[1]*Institute of Applied Physics, Darmstadt University of Technology, Hochschulstrasse 4a, 64289 Darmstadt, Germany*
[2]*Institute of Applied Physics, Westfälische-Wilhelms-Universität Münster, Corrensstrasse 2/4, 48149 Münster, Germany*
[3]*Institute of Physics, P.O. Box 57, 11001 Belgrade, Serbia*
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We display rich spatial and temporal dynamics of light fields counterpropagating in a saturable self-focusing medium numerically, and analyze instabilities that counterpropagating solitons experience. An expression for the maximum length that the medium must not exceed for the solitons to be stable is derived and connected to the coupling strength of beam interaction. The instability can lead to periodic or irregular temporal dynamics of the light beams. By considering mutually incoherent counterpropagating beams, we show that differences to the copropagating case are due to the different boundary conditions.

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One of the most active fields in recent research on spatial solitons [1] is the investigation of vector solitons [2], i.e., multicomponent light beams that jointly self-trap in nonlinear media. Although some exceptions exist [3], most of the work published on this topic treated the case where all beams forming the soliton propagate in the same direction. Recent papers [4–6] drew attention toward the possibility of creating solitons out of counterpropagating (CP) light beams. However, so far little attention has been paid to the temporal dynamics of the system.

CP light fields are also used for investigating pattern formation in nonlinear optical systems [7]. In these experiments the interference of CP fields and the resulting index grating in the medium play a key role. In this paper, however, we consider only the case of mutually incoherent CP light beams, and thus ignore such index gratings. Our focus will be on the qualitative changes brought about by the different boundary conditions compared to the copropagating case. Specifically, we show that CP fundamental beams cannot form a soliton if the medium is too long or the coupling strength too high, and that counterpropagation can lead to temporally dynamical states that can not be accessed by the usual steady-state treatments. A critical curve is identified in the plane (L,Γ) of control parameters, where L is the medium thickness and Γ the beam coupling strength, which separates steady-state soliton solutions from the spatially and/or temporally changing solutions.

We consider two mutually incoherent light fields in a nonlinear medium. In paraxial approximation the propagation of the beams is expressed by the set of equations

$$i\partial_z F(\rho,z) = -\nabla_\perp^2 F(\rho,z) + \Gamma E_0(\rho,z) F(\rho,z), \tag{1a}$$

$$-i\partial_z B(\rho,z) = -\nabla_\perp^2 B(\rho,z) + \Gamma E_0(\rho,z) B(\rho,z). \tag{1b}$$

Here F is the amplitude of the beam propagating in the positive z direction and B is the amplitude of the beam propagating in the opposite direction. The spatial coordinates in the plane orthogonal to the direction of propagation are denoted by ρ and $\nabla_\perp^2$ is the transverse Laplacian. For the CP beams one has split boundary conditions, i.e., the beam amplitudes $F(\rho,z=0)$ and $B(\rho,z=L)$ are specified at the opposite crystal faces. This is in contrast to the copropagating case, where the amplitudes of both beams are specified at $z=0$.

Γ measures the strength of the nonlinearity and E_0 is the nonlinear response of the medium. In the following we adopt a model that can be applied to metal vapors and that, although ignoring the nonlocality and anisotropy of the crystals, can be used as a simplified model for photorefractive media [8]. Then $\Gamma=(kn_0x_0)^2 r_{eff} E_e$, where k is the wave number of the laser used, n_0 is the refractive index of the unperturbed medium, r_{eff} is the effective coefficient of the electro-optic tensor, and E_e is the externally applied voltage needed for the screening effect to occur. Lengths in the transverse plane are scaled to x_0, usually a typical beam width, and in the z direction they are scaled to the diffraction length $L_D=2kx_0^2$. The temporal evolution of the nonlinearity is modeled by

$$\tau\partial_t E_0 + E_0 = -\frac{I}{1+I}, \tag{2}$$

τ being the relaxation time of the crystal and I is the light intensity $|F|^2+|B|^2$ scaled to I_d, which is the so-called dark intensity. In the following we set $x_0=10\ \mu\mathrm{m}$, $E_e=1.8$ kV/m, $r_{eff}=180$ pm/V, $n_0=2.35$, and $k=2\pi n_0/\lambda_0$, λ_0 being the laser wavelength (532 nm). This gives us $\Gamma=13.8$.

We are using the so-called isotropic model to describe the nonlinearity of the photorefractive crystal for the sake of simplicity. The role of the anisotropy [9] needs to be assessed for future investigations.

First we investigate the counterpropagation of two beams, each being the fundamental mode of the jointly induced waveguide, thus forming a soliton. As input we use two identical numerically calculated solitary beam profiles, with a maximum intensity of about $3I_d$ at the input faces of the crystal. Up to the length of the medium of $0.65L_D$ no sign of instability is observed, and both beams propagate through the jointly induced waveguide as solitons. However, at $L=0.68L_D$ the solitary solution becomes unstable. The result is shown in Fig. 1. At $t=25\tau$ the beams still propagate as solitons through their jointly induced waveguide. But the white noise included in the system excites an eigenmode that

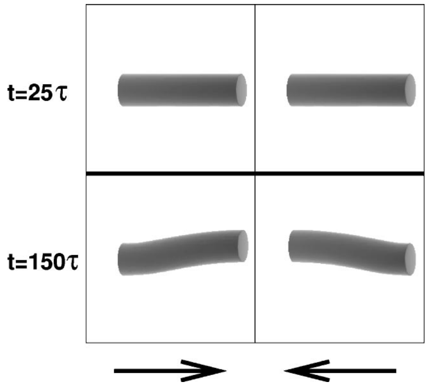

FIG. 1. Counterpropagating fundamental beams for a medium of length $L=0.68L_D$. The left column shows the forward and the right column the backward beam (the arrows indicate the direction of propagation). The top row is a snapshot after $t=25\tau$. Both beams propagate through the medium as solitons. At $t=150\tau$ (bottom row) the beams no longer propagate as solitons, but instead deviate on their way through the crystal.

grows in time and at $t=100\tau$ the beams no longer propagate as solitons, i.e., their intensity profiles change along the propagation direction. Both beams deviate inside the medium from the straight initial trajectories. As a consequence the beams no longer coincide at $z=0$ and $z=L$. Since the initial problem is rotationally symmetric, the direction into which the beams deviate is random. The intensity distribution at $t=150\tau$, presented in the bottom row of Fig. 1, shows a steady state of the system. Rotating this state by an arbitrary angle around the z axis also yields a steady state. Therefore the noise present in the system can randomly turn the state in one or the other direction. Any such rotation, however, takes place on a much slower time scale than the development of the instability.

Thus, the numerical results show that the length of the medium plays a significant role in the stability of CP solitons. Another important factor is the power of the beams. Decreasing the power of the beams stabilizes the CP solitons. However, if L is increased, the solitons become unstable again and change in a way similar to Fig. 1. In addition, if L is further increased, the beams do not reach steady state, but keep changing with time.

These results seem to contradict results obtained for the solitons in copropagating geometry. Two mutually incoherent solitons always attract each other, therefore one would expect that the two CP beams always form a stable soliton. To find an explanation for this instability, we consider the CP beams as particles whose motion along the z axis is subject to forces caused by the refractive index change in the medium [10]. Thus, we will only be concerned with the motion of the "center of mass" of the beams, $\mathbf{c}_1(z,t)$ and $\mathbf{c}_2(z,t)$. The center of mass of each beam will be attracted by the waveguide induced in the medium by the beams. Because the medium is noninstantaneous, we assume that the motion of $\mathbf{c}_i$ is determined by the light distribution a time τ ago. Furthermore, it is assumed that the force acting on $\mathbf{c}_i$ is proportional to the distance from the center of the waveguide. We thus arrive at a simple linear set of equations

$$\partial_{zz}\mathbf{c}_1(z,t)=K[\mathbf{c}_1(z,t-\tau)-\mathbf{c}_1(z,t)] + K[\mathbf{c}_2(z,t-\tau)-\mathbf{c}_1(z,t)], \tag{3a}$$

$$\partial_{zz}\mathbf{c}_2(z,t)=K[\mathbf{c}_1(z,t-\tau)-\mathbf{c}_2(z,t)] + K[\mathbf{c}_2(z,t-\tau)-\mathbf{c}_2(z,t)], \tag{3b}$$

where the constant K is determined by the strength of the nonlinearity and the power of the CP beams, and represents a measure for the mutual attraction of two beams. An approximate value of K is calculated below. To further simplify the problem, the separation of the centers of mass of two beams $\mathbf{d}(z,t)=\mathbf{c}_1(z,t)-\mathbf{c}_2(z,t)$ is introduced as one dynamical variable, and the center of mass of the system $\mathbf{C}(z,t)=[\mathbf{c}_1(z,t)+\mathbf{c}_2(z,t)]/2$ as the other. Furthermore, $\mathbf{c}_i(z,t-\tau)$ is replaced by $\mathbf{c}_i(z,t)-\partial_t\mathbf{c}_i(z,t)\tau$. Then the temporal evolution of the system can be described as

$$\mathbf{0}=-\partial_{zz}\mathbf{d}(z,t)-2K\mathbf{d}(z,t), \tag{4a}$$

$$2K\tau\partial_t\mathbf{C}(z,t)=-\partial_{zz}\mathbf{C}(z,t). \tag{4b}$$

(Note that there is no $\partial_t\mathbf{d}(z,t)$ term.)

We analyze the stability of the solution $\mathbf{d}(z,t)\equiv\mathbf{0}$ and $\mathbf{C}(z,t)\equiv\mathbf{0}$ using the tools of nonlinear dynamics. The following unstable eigenmode is identified as

$$\mathbf{d}(z,t)=\exp(\lambda t)\sin[\sqrt{2K}(z-L/2)]\mathbf{e}_{\rho}, \tag{5a}$$

$$\mathbf{C}(z,t)=A\exp(\lambda t)\cos[\sqrt{2K\tau\lambda}(z-L/2)]\mathbf{e}_{\rho}, \tag{5b}$$

where $\mathbf{e}_\rho$ is unit vector in the transverse plane. The constant A and the growth rate of the instability λ can be determined from the boundary conditions $\mathbf{c}_1(0,t)=\mathbf{c}_2(L,t)=0$ and $\partial_z\mathbf{c}_1(0,t)=\partial_z\mathbf{c}_2(L,t)=\mathbf{0}$. Using Eqs. (5) this translates into

$$2A\cos(\sqrt{2K\tau\lambda}L/2)=\sin(\sqrt{2K}L/2), \tag{6a}$$

$$2A\sqrt{\tau\lambda}\sin(\sqrt{2K\tau\lambda}L/2)=-\cos(\sqrt{2K}L/2). \tag{6b}$$

Discarding the unphysical case $\tau\lambda>1$ [in this case $\mathbf{c}_i(z,t-\tau)$ can no longer be replaced by $\mathbf{c}_i(z,t)-\partial_t\mathbf{c}_i(z,t)\tau$ as done in the above calculations] it can be shown that the solutions with positive λ (unstable eigenmodes) can only exist if $L>L_c$, where

$$L_c=\pi/\sqrt{2K}. \tag{7}$$

As can be seen from Eqs. (5) unstable eigenmodes exist for any orientation of the unit vector $\mathbf{e}_\rho$. These eigenmodes compete with each other, but only one can dominate and grow exponentially. In the real physical model described by Eqs. (1) and (2) higher-order effects become important for growing deviations from the beams' initial trajectory, thus leading to the steady state shown in Fig. 1.

To ascertain whether this simple criterion can serve as an estimate for predicting the onset of instability of CP solitons, one needs an estimate for K. To this end, we consider the case where the two beams that form the soliton are slightly shifted relative to each other, i.e., $F(\rho,z)=\psi(\rho+\epsilon\mathbf{e}_\rho)$ and

$B(\rho,z)=\psi(\rho-\epsilon\mathbf{e}_\rho)$. Here $\psi(\rho)$ is the solitary beam profile and ϵ measures the distance between the two beams. Inserting this into Eqs. (1), and assuming a steady state, we find

$$i\partial_z F=\beta F+\epsilon\Gamma(\mathbf{e}_\rho\cdot\boldsymbol{\nabla}_\perp)E_0F \tag{8}$$

and an analogous equation for B, where the real constant β is the propagation constant of the soliton. The last term on the right-hand side of Eq. (8) bends the beam towards the CP beam and also deforms it. Since we are only interested in the motion of the center of mass, we ignore this deformation. Thus, the bending of the beam is averaged over the transverse plane. From the motion of the center of mass we obtain the equation

$$\partial_{zz}\epsilon=\Gamma\frac{\int(\mathbf{e}_\rho\cdot\boldsymbol{\nabla}_\perp)^2E_0|\Psi|^2d\rho}{\int|\Psi|^2d\rho}\epsilon. \tag{9}$$

We thus have

$$K=-\frac{\Gamma}{2}\frac{\int(\mathbf{e}_\rho\cdot\boldsymbol{\nabla}_\perp)^2E_0|\Psi|^2d\rho}{\int|\Psi|^2d\rho}. \tag{10}$$

Inserting the value of K thus obtained into Eq. (7) we find that $L_c=0.84L_D$ for the soliton in Fig. 1, which is longer than the numerically determined stability threshold of about $0.68L_D$. Nonetheless, this is still reasonably close, considering the crude approximations used. In addition, Eqs. (7) and (10) can explain the fact that solitons with lower intensity can be stable in longer media because they have a lower value of K. This is due to the fact that the refractive index change induced by weaker beams is smaller, and therefore the waveguides are not as attracting.

Similarly, Eqs. (7) and (10) can explain why increasing the strength of the nonlinearity destabilizes CP solitons. Increasing the strength of the nonlinearity, for example, by increasing the externally applied voltage E_e means increasing the value of Γ. If we consider the length of the medium L fixed and instead allow K to vary, Eq. (7) can be written as $K_c=\pi^2/2L^2$, where K_c is the minimum value of K needed to obtain an unstable eigenmode in the system of model Eqs. (3). Using Eq. (10) this can be translated into an equation for a critical value of Γ where the instability sets in

$$\Gamma_c=-\frac{\pi^2}{L^2}\frac{\int|\Psi|^2d\rho}{\int(\mathbf{e}_\rho\cdot\boldsymbol{\nabla}_\perp)^2E_0|\Psi|^2d\rho}. \tag{11}$$

Note that the quotient of the integrals on the right-hand side scales as $1/\Gamma$ for solitons with the same maximum intensity, hence in the (L,Γ) parameter plane the stable and unstable configurations are, according to Eq. (7), separated by the critical line $\Gamma L=\text{const}$. For solitons with a maximum intensity of $0.9I_d$ in each component we calculate $\Gamma_c=16.9$ for a medium of length $L=0.8L_D$. Numerically we found that the solitons become unstable if Γ is bigger than 13.1. Equations (7) and (10) thus can not be used as an exact criterion to predict the stability properties of CP solitons, but can serve as an estimate where to look for the critical length of the medium where the solitons become unstable. Furthermore, they give an insight into the mechanisms which cause the instability.

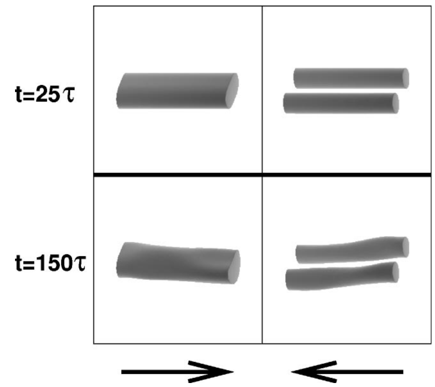

FIG. 2. Counterpropagating fundamental and dipole beams for a medium length of $L=0.82L_D$. As in Fig. 1, the beams initially propagate as solitons (upper row). When the instability sets in, the light distribution is no longer symmetric to the dipole axis (bottom row).

In the next step we investigate the stability of solitons consisting of a fundamental and a dipole mode. Dipole-mode vector solitons are well studied in copropagating geometry, and are known to be very robust [11–13]. As in the case of two CP fundamental modes, the simulations show that the beams no longer propagate as solitons through the medium, if the medium length exceeds a certain value. In Fig. 2 we present a case where the medium length is slightly above that critical value. As already seen in Fig. 1, the beams deviate from their initial trajectories as they propagate. It is interesting to notice that the deviation occurs in a direction perpendicular to the plane of the dipole. This fact can be explained by reconsidering the arguments that lead to the set of Eqs. (3). There we considered the case of two CP rotationally symmetric solitons. Because of the rotational symmetry, the value of K is independent of the direction into which the beams deviate. The problem of a CP fundamental and dipole beam, however, is not rotationally symmetric. Numerical calculations show that the two beams attract each other more strongly when they deviate perpendicular to the dipole plane than when they deviate parallel to the plane. This leads to a higher effective value of K for deviations perpendicular to the dipole plane, and therefore to a shorter critical length L_c, according to the estimate, Eq. (7). Therefore only two stable steady states of the system exist: the one shown in the bottom part of Fig. 2 and the other one, where both beams deviate by the same amount in the opposite direction.

The following step is to investigate the counterpropagation of a fundamental beam and a vortex. We found that for short values of L, used so far, there is no deviation of the beams during their propagation through the medium. But, as could be expected from the analogy to the copropagating

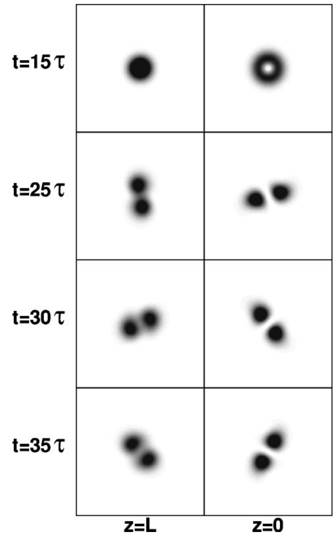

FIG. 3. Temporal evolution of a fundamental beam and a CP vortex for $L=1.1L_D$. After a certain time, the vortex breaks up into a dipole that rotates with time. The fundamental beam splits into two beamlets that corotate with the dipole. The left column shows the fundamental beam, the right column the vortex.

case, the vortex breaks up into a dipole beam during propagation, if the medium is long enough. Moreover, this system does not possess a steady state. An example of the temporal dynamics is shown in Fig. 3. Here $L=1.1L_D$. At $t=15\tau$ the vortex has not yet broken up into a dipole. At $t=25\tau$ however, the vortex beam incident upon the crystal leaves the crystal as a dipole. This also leads to a deformation of the fundamental beam. Note that the dipole and the fundamental beam are not aligned in the figure, because the two beams are shown at two different positions in the crystal, $z=0$ and $z=L$, respectively. The dipole and the fundamental beam do rotate with time. In the simulations the rotation continues indefinitely (we stopped the simulations at $t=200\tau$). This rotation, therefore, represents a periodic dynamic state of the system.

In the final step we examined the case of two CP vortices. In this case the vortices breakup as well, surprisingly not into

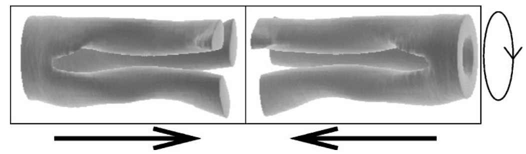

FIG. 4. Isosurface plot of two CP vortices. Breakup into three rotating beamlets is visible. Because of the topological charge, the beamlets start to spiral, which is only weakly visible due to the short propagation distance. The beamlets rotate in the direction indicated by the arrow on the right.

two beamlets, as could be expected, but into three beamlets, as shown in Fig. 4. They thus strongly remind of rotating soliton clusters investigated in Ref. [14]. As in the case of the counterpropagation of a fundamental and a vortex beam, the beamlets start to rotate at the exit faces of the crystal. The vortices' diameter must, however, be chosen carefully for the rotation to occur. In this case we observe rotation of the beamlets for times up to $t=80\tau$. This time corresponds to more than five round trips. After that the motion of the beamlets becomes irregular.

In summary, we have shown that because of the different boundary conditions, solitons in photorefractive media behave very differently in counterpropagating geometry than in copropagating geometry. If the medium exceeds a certain length, or the coupling exceeds a certain strength, the CP solitons consisting of two fundamental beams become unstable. It has been shown that this effect can be explained by considering a very simple model describing the motion of the centers of mass of beams. Furthermore, the counterpropagation of a fundamental and a vortex beam leads to a temporally periodic dynamical state of the system. The counterpropagation of two vortices leads to rich dynamic behavior that can be periodic, as well as irregular.

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