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Self-trapped bidirectional waveguides in a saturable photorefractive medium

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A time-dependent model for the generation of joint waveguides by counterpropagating light beams in photorefractive crystals is introduced. Depending on initial conditions and parameter values, the beams form stable structures or display periodic and irregular dynamics. Steady-state solutions nonuniform in the direction of propagation are found, representing a general class of self-trapped waveguides that include counterpropagating spatial vector solitons as a particular case.

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During the last decade spatial screening solitons [1] have been considered almost exclusively in copropagation geometry. Recent progress in generating optical solitons consisting of counterpropagating (CP) fields by Cohen *et al.* [2] has renewed interest in CP wave mixing, extensively studied in the past [3–5]. However, such geometries in photorefractive (PR) media are prone to instabilities [6–8] and are often employed for transverse optical pattern formation [9]. In particular, *temporal* instabilities were shown to result in self-oscillation, chaos, and bistability [3,4]. It is therefore of importance to investigate the temporal behavior of CP self-trapped beams in PR crystals with finite response time. Furthermore, one may easily envision interest in a stable self-adjustable connection of two arrays of beams across a PR crystal.

In this Rapid Communication we derive equations for the propagation of beams, similar to the bimodal CP solitons in Kerr media [5], and collisions of screening PR solitons propagating in opposite directions [2]. We formulate a time-relaxation procedure for the determination of space charge field and refractive index modulation in PR crystals. Dynamical effects are found important for understanding the behavior of CP beams. We display numerically the temporal formation of bright spatial screening vector solitons formed by CP beams, and discuss their interactions in $(1+1)$ spatial dimensions. Beyond soliton solutions, we introduce a more general class of steady-state induced waveguides. Additionally, a situation where the interacting beams do not converge to a stationary structure, but alternate between different states, is reported.

We consider two CP light beams in a PR crystal, in the paraxial approximation, under conditions suitable for the formation of screening solitons. The optical field is given as the sum of CP waves $F\exp(ikz+i\omega t)+B\exp(-ikz+i\omega t)$, k being the wave vector in the medium, F and B are slowly varying envelopes of the beams. The light intensity I is measured in units of the background light intensity, also necessary for the generation of solitons. After averaging in time on the scale of response time τ_0 of the PR crystal, the total intensity is given by

$$1+I=(1+I_0)\{1+\varepsilon[m\exp(2ikz)+\text{c.c.}]/2\}, \qquad (1)$$

where $I_0=|F|^2+|B|^2$ and $m=2FB^*/(1+I_0)$ is the modulation depth. Here the parameter ε measures the degree of *temporal* coherence of the beams related to the crystal relaxation time: for $\varepsilon=0$, i.e., when the relative phase of the beams varies much faster than τ_0, the beams are effectively incoherent. In the opposite case $\varepsilon=1$, the intensity distribution contains an interference term that is periodically modulated in the direction of propagation z, chosen to be perpendicular to the c axis of the crystal, which is also the x axis of the coordinate system. Beams are polarized in the x direction, and the external electric field E_e, necessary for the formation of self-trapped beams, also points in the x direction. The electric field in the crystal couples to the electro-optic tensor, giving rise to a change in the index of refraction

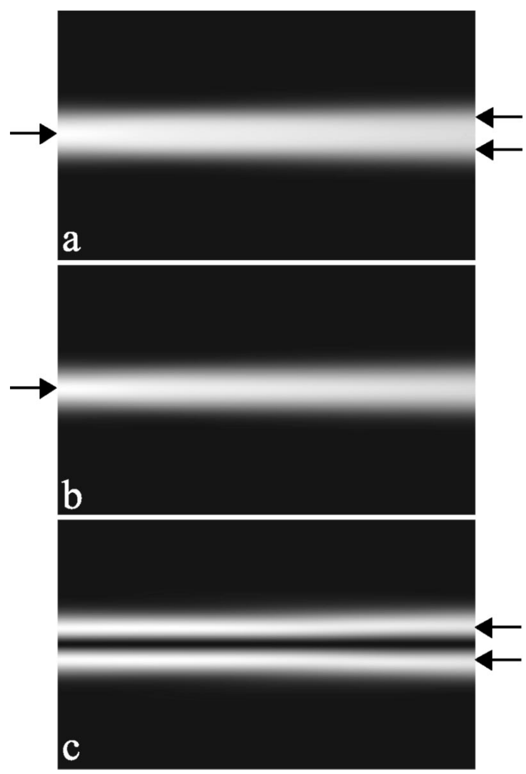

FIG. 1. Counterpropagating dipole-mode vector soliton (a), made out of a fundamental beam propagating to the right (b), and a coherent dipole beam propagating to the left (c). Coupling strength $\Gamma=3.3$.

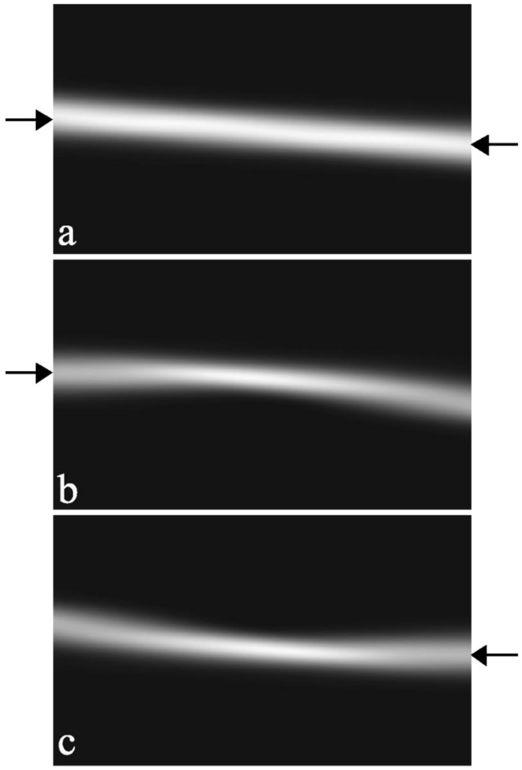

FIG. 2. Bidirectional waveguide. (a) Total intensity distribution; (b) right-propagating and (c) left-propagating beams. Layout as in Fig. 1, parameters $\varepsilon=1$, and $\Gamma=5$. Initial peak intensities $I_F=I_B=1$.

of the form $\Delta n=-n_0^3 r_{eff}E/2$, where n_0 is the unperturbed index, r_{eff} is the effective component of the electro-optic tensor, and E is the x component of the total electric field. It consists of the external field and the space charge field E_{sc} generated in the crystal, $E=E_e+E_{sc}$.

The intensity modulates the space charge field, which we represent in the normalized form

$$E_{sc}/E_e=E_0+\frac{1}{2}[E_1\exp(2ikz)+\text{c.c.}], \tag{2}$$

where E_0 is the homogeneous part of the x component of the space charge field, and $E_1(x,z)$ is the *slowly varying* part of the space charge field, proportional to ε. It is E_0 that screens the external field, and E_1 is the result of the interference pattern along the z direction.

In the isotropic approach, one assumes a local approximation to the space charge field, and looks for a solution with saturable nonlinearity $E=E_e/(1+I)$. Substituting Eqs. (1) and (2) in this expression, neglecting higher harmonics and terms quadratic in m, we obtain as a steady-state solution

$$E_0=-\frac{I_0}{1+I_0},\quad E_1=-\frac{\varepsilon m}{1+I_0}. \tag{3}$$

Temporal evolution of the space charge field is introduced by assuming relaxation-type dynamics [11]

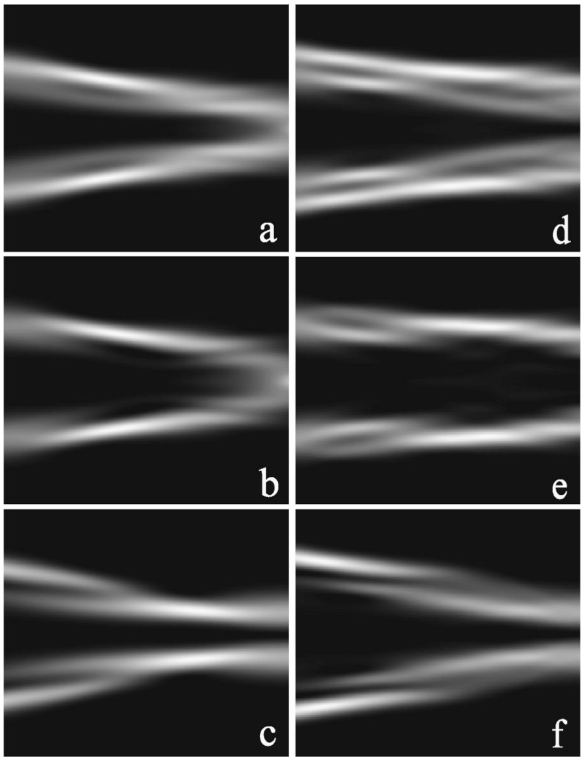

FIG. 3. Incoherent (a)–(c) and coherent (d)–(f) interaction of two pairs of CP beams. The initial offset is $4x_0$ for in-phase beams propagating to the right in (b) and (e), and $2x_0$ for the out-of-phase beams propagating to the left in (c) and (f). Parameters and layout as in Fig. 1.

$$\tau\partial_t E_0+E_0=-\frac{I_0}{1+I_0}, \tag{4a}$$

$$\tau\partial_t E_1+E_1=-\frac{\varepsilon m}{1+I_0}, \tag{4b}$$

where the relaxation time of the crystal τ is inversely proportional to the total intensity $\tau=\tau_0/(1+I)$, i.e., illuminated regions in the crystal react faster. The assumed dynamics is that the space charge field builds up towards the steady state, which depends on the light distribution, which in turn is slaved to the slow change of the space charge field. As will be seen later, this does not preclude a more complicated dynamical behavior.

Selecting synchronous terms in the nonlinear paraxial wave equation, we obtain the propagation equations

$$i\partial_z F+\partial_x^2 F=\Gamma[E_0F+E_1B/2], \tag{5a}$$

$$-i\partial_z B+\partial_x^2 B=\Gamma[E_0B+E_1^*F/2], \tag{5b}$$

where the parameter $\Gamma=(kn_0x_0)^2 r_{eff}E_e$ represents the coupling strength, and the rescaling $x\to x/x_0$, $z\to z/L_D$, $(F,B)\to(F,B)\exp(-i\Gamma z)$ is used. Here x_0 is the typical beam waist and $L_D=2kx_0^2$ is the diffraction length [10]. Propagation equations are solved numerically, concurrently with the temporal equations. The numerical procedure consists in solving

Eqs. (4) for the components of the space charge field, with the light fields obtained at every step as *guided modes* of the induced common waveguide. It is described in Refs. [8,12].

The procedure starts with the given initial light fields at the left and right crystal faces, and the space charge field set to zero. Within each temporal step an iterative relaxation procedure, based on a split-step beam propagation method, is applied to the propagation equations with the given space charge field. In each iterative step the beams are propagated simultaneously from their input faces, using the value from the previous iteration for the other beam. The propagation equations are treated until spatial convergence is achieved, and the converged intensities are used in the next temporal step, to update the space charge field and the crystal relaxation time. Both loops are iterated until convergence, which, however, is not necessarily reached in the temporal loop. In that case a *dynamical* state is obtained.

Head-on collision of the beams with initial soliton profiles, after temporal relaxation to a steady state, results in the formation of a CP soliton (not shown), similar to the one found in Ref. [2]. One can easily generalize this approach, introducing higher-order CP solitons, similar to the multihump vector solitons in copropagating geometry (see, e.g., Ref. [13]). In Fig. 1 we present a particular case of a dipole-mode CP soliton. Dipole beam is launched from the right, and a power-matched single beam from the left. Such a bimodal CP soliton has been studied in Ref. [5]. The size of data windows in all figures is 10 beam diameters transversely by 2 diffraction lengths longitudinally.

Shooting initial beams with arbitrary parameters generally leads to z dependent or nonstationary character of the beam propagation. In some domain of the initial parameters, for example, with the relative angle of beam scattering θ close to π and small initial transverse offset, our time-relaxation procedure converges to *stationary in time* structures, which we denote as steady-state *self-trapped waveguides* [14]. The formation of a single bidirectional waveguide is shown in Fig. 2. Two coherent Gaussian beams are launched at different lateral positions perpendicular to the crystal edges, $\theta=\pi$. Both beams diffract initially, until the space charge field is developed in time to form the waveguide induced by the total light intensity, Fig. 2(a), and this induced waveguide traps both beams, Figs. 2(b) and 2(c). When the initial separation is four or more beam diameters, the beams hardly feel the presence of each other, and focus into individual solitons. For the separation of two beam diameters, the interaction is strong enough for the beams to form a joint waveguiding structure, as is shown in Fig. 2.

We next examine the difference between the coherent and incoherent interaction of beams. Two steady-state solutions with the same boundary conditions but for different degrees of mutual coherence ε are shown in Fig. 3. Counterpropagating beam components made of two pairs of beams are launched with a lateral offset. The beams to the right are in phase, and aim at the center of the opposite crystal face. The beams to the left are out of phase, and launched in parallel. Figures 3(a)–3(c) depict the incoherent interaction, $\varepsilon=0$. The beams attract, focus and overlap tightly, but the ones to the right (b) are still capable of building the intense spot in between the other two. However, in the coherent case $\varepsilon=1$, shown in Figs. 3(d)–3(f), beams focus and overlap less, and the beams to the right (e) are expelled from the region between the other beams. Also, the time scale of buildup dynamics is shorter for the coherent interaction of beams.

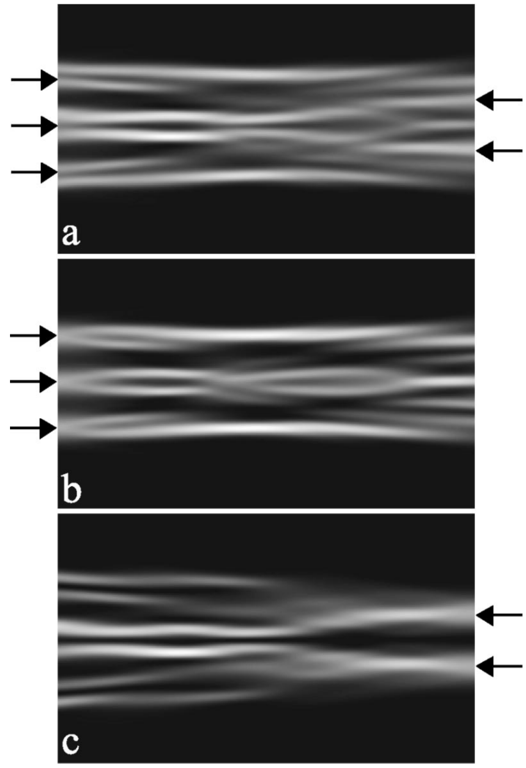

FIG. 4. Unstable self-organized beam structure after $116\tau_0$ temporal steps, at the moment of symmetry breaking. Three in-phase beams propagate to the right (b), two out-of-phase beams to the left (c). The components have equal powers and the coupling $\Gamma=10$. Other parameters as in Fig. 1.

We would like to note here that for propagation distances exceeding some threshold value, i.e., for larger crystal lengths, and for increasing coupling strengths we observe modulational instabilities developing in time, even for the initial beams corresponding to the exact steady state solitons. In that case *dynamical* states follow. Modulational instability is a topic of ongoing research and beyond the frame of present paper.

Of special interest are those self-trapped structures that dynamically do not converge to a steady state. Such structures represent time-dependent, as well as z dependent, waveguides that cannot be described by the usual steady-state theory of spatial solitons. Whereas the z dependence can be ascribed to the general definition of longitudinal waveguide modes, the time dependence is an important feature, caused by the slow response of PR crystals. An example is depicted in Fig. 4, where a collision of three against two power-matched coherent beams is presented. The initial configuration is such that the three beams propagating to the right interfere constructively (b), to overlap with the two CP out-of-phase beams (c). During the time evolution of this dynamical state we have observed several alternations of transversely symmetrical structures, similar to the one shown

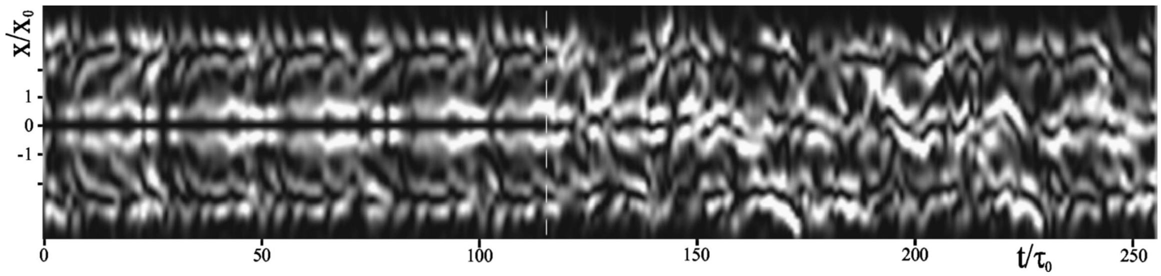

FIG. 5. Temporal evolution of the output intensity distribution of the two-lobe left-propagating beam at the left face of the crystal. Dashed line at $t=116\tau_0$ shows the slice corresponding to Fig. 4(c), where the modulational instability breaks the transverse symmetry.

in Fig. 4, and identified such behavior as a quasiperiodic self-oscillation [3], clearly seen in Fig. 5 for $t<116\tau_0$. At that point the development of *transverse* symmetry-breaking instability is observed, which results in irregular dynamics, shown in Fig. 5 for $t>116\tau_0$.

In conclusion, we have developed a theory of self-trapped bidirectional waveguides. In CP geometry, the inclusion of time-dependent effects was found to be crucial for the formation of dynamical waveguiding structures. We demonstrated the generation of a counterpropagating (1+1)D vector soliton numerically and proposed a more general class of nonsoliton steady-state solutions. The level of temporal coherence of interacting beams influences the mutual coupling due to the formation of a refractive index grating. In addition to the generation of steady-state induced waveguides, the dynamic alternation of states followed by a transverse modulational instability, as well as the onset of longitudinal modulational instabilities were observed.

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