

Exact solution to photorefractive and photochromic two-wave mixing with arbitrary dependence on fringe modulation

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An exact solution to the slowly varying envelope equations for two-wave mixing with both photorefractive and photochromic gratings present and with an arbitrary dependence of the gain and absorption on the fringe modulation is obtained. © 1996 Optical Society of America

When a two-wave laser interference pattern illuminates a photorefractive crystal, two kinds of grating are generated in the crystal, photorefractive (PR) gratings induced in the refractive index and photochromic (PC) gratings induced in the absorption coefficient. Most attention is focused on the PR gratings, since by virtue of their $\pi/2$ shift with respect to the interference pattern they cause a variety of interesting phenomena. Absorption gratings, being exactly in phase (or out of phase) with the fringe pattern, cause increased absorption and as such are generally avoided.¹ However, it was recently realized that a combined action of the two gratings can lead to enhanced diffraction efficiency in some sillenite PR crystals.² It was also noted that, curiously, two-wave mixing with pure PC gratings in the transmission geometry (TG) is formally equivalent to two-wave mixing with pure PR gratings in the reflection geometry³ (RG). And last, but not least, it was realized that a complete description of two-wave mixing in PR crystals with strong coupling and depleted pump requires the inclusion of both the absorption and the dependence of couplings on the fringe modulation depth.

I provide here an exact solution of the two-wave mixing equations with both kinds of grating present and with an arbitrary dependence of the coupling constant and the absorption coefficient on the modulation depth.

We proceed right to the crux of the problem—the solution of two-wave mixing equations in the slowly varying envelope approximation³:

$$\begin{aligned} I_1' &= -\alpha_0 I_1 - \frac{\Gamma + \alpha}{I} I_1 I_2, \\ \sigma I_2' &= -\alpha_0 I_2 + \frac{\Gamma - \alpha}{I} I_1 I_2. \end{aligned} \quad (1)$$

Here I_1 and I_2 are the intensities of copropagating or counterpropagating laser beams mixing in the crystal, α_0 is the linear absorption, Γ is the coupling coefficient to the PR phase gratings, and α is the coupling coefficient to the absorptive PC gratings. I is the total intensity, and $\sigma = \pm 1$ is the parameter controlling the geometry of mixing. The prime stands for the derivative along the propagation direction; hence for $\sigma = +1$ the transmission (copropagating) geometry is in force, whereas for $\sigma = -1$ the reflection (counterpropagat-

ing) geometry is in force. We treat these geometries equally.

In writing Eqs. (1) a $\pi/2$ phase shift between the intensity interference fringes and the refractive-index gratings is assumed (as is usual for PR gratings), whereas a zero phase shift is assumed for the absorptive gratings (also usual for PC gratings). These assumptions simplify the problem of phases of the interacting beams (making them constant throughout the crystal). An ingredient that makes the problem of solving these equations more interesting is the allowance for functional dependence of the parameters Γ and α on fringe modulation. As is known,³ these parameters are constant only in the lowest-order (Kukhtarev) approximation, in which the space-charge field responsible for the appearance of gratings is directly proportional to the fringe modulation $m = 2(I_1 I_2)^{1/2}/(I_1 + I_2)$. The Kukhtarev approximation⁴ is valid for small modulation depth. For arbitrary modulation depth, Γ and α should be considered functions of m , and an accepted form of presenting these functions is

$$\Gamma(m) = \Gamma_s \frac{f_x(m)}{m}, \quad \alpha(m) = \alpha_s \frac{f_x(m)}{m}, \quad (2)$$

where Γ_s and α_s are the saturation values of the coupling constant and absorption and $f_x(m)$ is the model function introduced by Refregier *et al.*⁵ to account for the deviation of the $\Gamma(m)$ dependence from a straight line. Even though the forms of $\Gamma(m)$ and $\alpha(m)$ suggest a common physical origin for both gratings, this need not be so, and the method of solution allows for an arbitrary form of these functions. A number of forms for the model function are currently in use ($x = 1, 2, 3, 4$, starting with $f_1 = m$ for the Kukhtarev approximation)⁵⁻⁷; however, here we are not concerned much with the form. Although these functions are important for the phenomenological description of the physics of PR effect, in the solution of wave equations they play no role. The equations can be solved exactly for any form of the model function. The solutions of Eqs. (1) proceed along the following leins.⁸

We first treat the TG, since as an initial-value problem (both fields given on the $z = 0$ face of the crystal) it is a bit simpler. The form of equations

suggests a change of variables:

$$f = 2(I_1 I_2)^{1/2}, \quad F = \frac{1}{2} \ln\left(\frac{I_1}{I_2}\right). \quad (3)$$

In the new variables the fringe modulation becomes a function of only one variable, $m = \text{sech}(F)$. Even more importantly, the equations separate:

$$2F' = \alpha \tanh(F) - \Gamma, \quad \frac{2f'}{f} = \Gamma \tanh(F) - \beta, \quad (4)$$

where $\beta = 2\alpha_0 + \alpha$. These equations present a set of quadratures and as such are easily solved. Thus, for an arbitrary dependence of Γ and α on m , one solves the integral

$$z = \int_{F_0}^F \frac{2dF}{\alpha(m)\tanh(F) - \Gamma(m)} \quad (5)$$

to obtain an implicit function $z(F, F_0)$. Different models lead to different functions. Not all models lead to integrals that can be expressed in closed form. For the Kukhtarev approximation the integration is easy:

$$z = \frac{2}{\alpha_s^2 - \Gamma_s^2} \left\{ \Gamma_s(F - F_0) + \alpha_s \ln\left[\frac{\cosh(F - F_c)}{\cosh(F_0 - F_c)} \right] \right\}, \quad (6a)$$

where $F_c = \tanh^{-1}(\alpha_s/\Gamma_s)$ and it is assumed that $\Gamma_s > \alpha_s$. Regardless of the form of the model function $f_x(m)$, a universal function $f(z)$ is found:

$$f = f_0 \exp\left[\frac{\Gamma_s}{\alpha_s} (F - F_0) + \left(\frac{\Gamma_s^2 - \alpha_s^2}{2\alpha_s} - \alpha_0 \right) z \right]. \quad (6b)$$

The values of the integration constants F_0 and f_0 are given in terms of the initial values for the intensities $I_1(z = 0) = C_1$ and $I_2(z = 0) = C_2$, according to Eqs. (3). Once the functions f and F are known, the intensities are given by

$$I_1 = \frac{f}{2} \exp(F), \quad I_2 = \frac{f}{2} \exp(-F). \quad (7)$$

Before presenting some results, let us go through the RG procedure. Making the same set of variable transformations, one obtains the following integrals for F and f :

$$2F' = \Gamma \tanh(F) - \beta, \quad \frac{2f'}{f} = \alpha \tanh(F) - \Gamma. \quad (8)$$

Again, one solves the integral for F :

$$z(F, F_0) = \int_{F_0}^F \frac{2dF}{\Gamma(m)\tanh(F) - \beta(m)}, \quad (9)$$

which, for constant Γ and α , becomes

$$z = \frac{2}{\Gamma_s^2 - \beta_s^2} \left[\beta_s(F - F_0) + \Gamma_s \ln \left| \frac{\sinh(F - F_c)}{\sinh(F_0 - F_c)} \right| \right], \quad (10a)$$

where $\beta_s = 2\alpha_0 + \alpha_s$ and $F_c = \tanh^{-1}(\beta_s/\Gamma_s)$ and it is assumed that $\Gamma_s > \beta_s$. Similarly, one obtains for f

$$f = f_0 \exp\left[\frac{\alpha_s}{\Gamma_s} (F - F_0 + \alpha_0 z) + \left(\frac{\alpha_s^2 - \Gamma_s^2}{2\Gamma_s} \right) z \right], \quad (10b)$$

where, at one point, it is assumed that $\alpha_0/\Gamma \approx \alpha_0/\Gamma_s$.

The only complication is that in the RG the integration constants F_0 and f_0 are not so readily obtained as in the TG. Now we have two-point boundary values for the intensities $I_1(z = 0) = C_1$ and $I_2(z = d) = C_2$, where d is the thickness of the crystal. One finds f_0 from the relation $f_0 = 2C_1 \exp(-F_0)$, and F_0 is found by solution of the following algebraic equation:

$$\beta_s(F_0 - F_d) + \frac{\Gamma_s^2 - \beta_s^2}{2} d = \Gamma_s \ln \left| \frac{\sinh(F_d - F_c)}{\sinh(F_0 - F_c)} \right|, \quad (11)$$

where F_d is given by

$$F_d = \frac{(\delta + 1)F_0 - \left(\delta\alpha_0 + \frac{\delta^2 - 1}{2} \Gamma_s \right) d + \ln\left(\frac{C_2}{C_1}\right)}{\delta - 1}, \quad (12)$$

where $\delta = \alpha_s/\Gamma_s$. This solution must be performed numerically. Similarly to the previous use of the method,⁸ Eq. (11) allows for more than one solution (three); however, only the solution located between the two singular points on the right-hand side should be retained. This completes the RG solution procedure.

I now discuss some of the consequences of the solutions found. Given the simple explicit expressions for intensities, one can derive interesting relations for the quantities of experimental interest. For example, the transmissivities in the TG along the directions of beams 1 and 2 are given by

$$T_1 = \frac{I_{1d}}{C_1} = C \exp\left[\frac{1 + \delta}{\delta} (F_d - F_0) \right],$$

$$T_2 = \frac{I_{2d}}{C_2} = C \exp\left[\frac{1 - \delta}{\delta} (F_d - F_0) \right], \quad (13)$$

where $C = \exp[(\Gamma_s^2 - \alpha_s^2 - 2\alpha_0\alpha_s)d/2\alpha_s]$. Hence T_1 is proportional to T_2^ρ , where $\rho = (1 + \delta)/(1 - \delta)$. Likewise, in the RG one finds

$$T_1 = \frac{I_{1d}}{C_1} = C \exp[(1 + \delta)(F_d - F_0)],$$

$$T_2 = \frac{I_{2d}}{C_2} = C^{-1} \exp[(1 - \delta)(F_d - F_0)], \quad (14)$$

where now $C = \exp[(\alpha_s^2 - \Gamma_s^2 + 2\alpha_0\alpha_s)d/2\Gamma_s]$. Hence the same relation as in the TG holds.

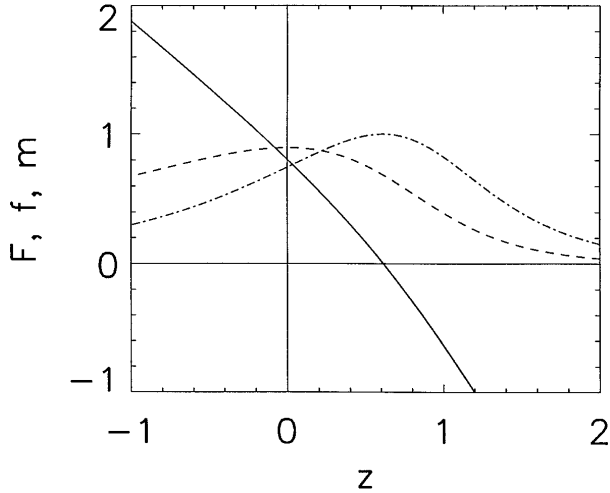


Fig. 1. Functions F (solid curve), f (dashed curve), and m (dashed-dotted curve) as functions of z in the TG.

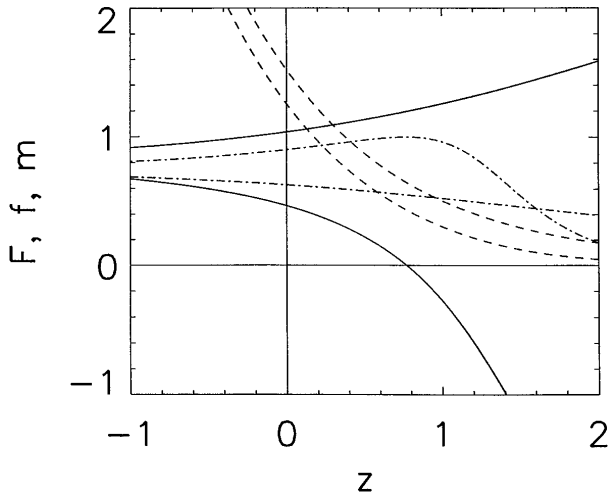


Fig. 2. Functions F (solid curves), f (dashed curves), and m (dashed-dotted curves) as functions of z in the RG. Two branches are shown for each function; however, only the lower branches should be taken into account.

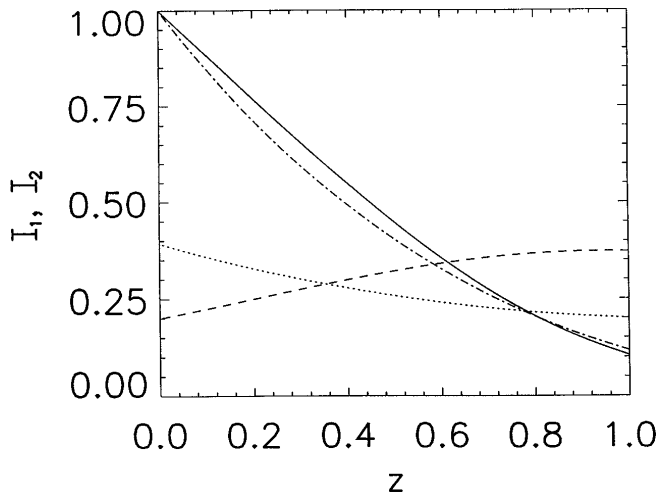


Fig. 3. Intensities I_1 and I_2 as functions of z , for both the TG and the RG, corresponding to the functions F and f presented in Figs. 1 and 2. The solid and dashed-dotted curves are I_1 for TG and RG, and the dashed and the dotted curves are I_2 for TG and RG.

Next, consider the case $\alpha_0 = 0$. Looking at Eqs. (6) and (10), one notes a high degree of symmetry. As mentioned above, these expressions are written for the case $\Gamma_s > \alpha_s$, which seems to be more relevant. The case $\alpha_s > \Gamma_s$ is described by the same expressions, but the cosh and the sinh functions should be interchanged. Hence, one can formally go from TG to RG by interchanging Γ_s and α_s . Thus a pure PR case in TG is equivalent to the pure PC case in RG and vice versa. These facts were first noted in Ref. 3. The presence of linear absorption breaks this symmetry. Also, the inclusion of boundary conditions breaks the symmetry.

Some of the results are presented in Figs. 1–3. The functions $F(z)$, $f(z)$, and $m(z)$ are shown for a typical set of parameters: $C_1 = 1$ (the unit of intensity), $C_2 = 0.2$, $\Gamma_s = 3$, $\alpha_s = 1$, $\alpha_0 = 0.5$ (in cm^{-1}), and $d = 1$ cm, in both the TG and the RG and for the Kukhtarev approximation. Qualitatively similar behavior is observed in all the models. Figure 1 presents the three functions in the TG. Even though the graphs are drawn for both positive and negative z , only the interval between $z = 0$ and $z = d = 1$ is physically relevant. Figure 2 presents the corresponding situation in the RG. Multistability is observed, with two branches for the function $F(z)$ [the function $z(F)$ is well behaved] and the two corresponding functions for $f(z)$ and $m(z)$. However, only the lower branch of $F(z)$ is allowed (the one below F_c), since then F_0 is smaller than F_c , as required. Here $F_c \approx 0.805$ and $F_0 \approx 0.467$. The allowed functions $f(z)$ and $m(z)$ are also the lower branches. One sees that, even though the Kukhtarev approximation is assumed, m is not small. This means that such an approximation is not entirely self-consistent and that it is better to use the complete model, with the provision for the m dependence. Figure 3 depicts the corresponding intensities I_1 and I_2 .

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