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Transverse effects in double phase conjugation

M.R. Belić [a,b], J. Leonardy [a], D. Timotijević [b], F. Kaiser [a]

[a] *Institute of Applied Physics – Nonlinear Dynamics, Technical University Darmstadt, Hochschulstr. 4a, 64289 Darmstadt, Germany*

[b] *Institute of Physics, P.O.Box 57, 11001 Belgrade, Yugoslavia*

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Abstract

Spatial and dynamical effects in double phase conjugate mirrors are investigated analytically and numerically. A variety of spatial and temporal effects are observed, such as beam-bending, self-defocusing, mode oscillations and irregular pattern formation. We conclude that in more than one spatial dimension the double phase conjugate mirror is a convective oscillator. For strong beam couplings and strong diffraction we find that the oscillation threshold is not well defined, and that the double phase conjugate mirror becomes unstable. An improved agreement with experimental results is obtained.

Photorefractive (PR) oscillators or self pumped phase conjugate mirrors (PCM) are essential parts of any envisioned device employing optical phase conjugation (OPC) [1]. Experimental realization of PR oscillators is not very difficult, understanding their behavior is. An especially troublesome aspect of their behavior are inherent instabilities. Another problem is poor understanding of transverse effects and mode coupling in these oscillators.

An interesting geometry from the applications point of view is the double phase conjugate mirror (DPCM) (Fig. 1). Assuming the input waves to be mutually incoherent (no reflection gratings), the beam A_1 is phase coherent with A_4 and the beam A_3 is phase coherent with A_2. Thus, they can pairwise build transmission-type gratings and the PC signal can be amplified at the expense of the pump even though the pump and the probe are incoherent. This arrangement is very convenient for remote operation, for example communication through the atmosphere or through fibers.

DPCM was shrouded in controversy almost from the conception. The possibility of DPCM was suggested by Cronin-Golomb et al. [1], but was deemed improbable by the same group, because of the competing conical emissions. It took a few years for Fischer and his group [2] to demonstrate the feasibility and to establish the operation of DPCM. What made it possible is the preferential amplification of both conjugate beams by a particular set of fanning gratings.

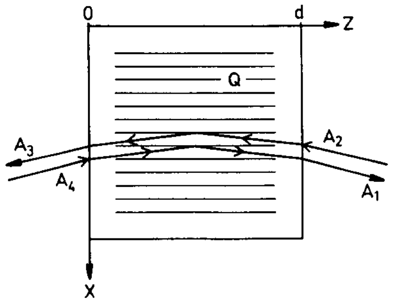

Fig. 1. Double-phase conjugate mirror. The pump beams A_2, A_4 enter the crystal from the opposite sides. A_1 is the PC of A_2 and A_3 is the PC of A_4. The thickness of the crystal d is set to 1 cm.

A more recent controversy surrounding DPCM is

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concerned with the question whether DPCM is an oscillator or an optical amplifier. Plane-wave analysis suggests that DPCM is indeed an oscillator [3]. In the models with two and three spatial dimensions, however, this is not clear. The theory presented in [4] claims that DPCM is still an oscillator, while the theoretical and experimental analysis of [5] shows that DPCM is a convective amplifier. We believe that DPCM is a convective oscillator.

To understand the operation of DPCM, we study its spatio-temporal behavior. The basic equations describing four-wave mixing (4WM) processes in paraxial approximation are of the form:

$$\partial_z A_1 + \mathrm{i} f \Delta A_1 - \alpha A_1 = Q A_4, \tag{1}$$

$$\partial_z A_3 - \mathrm{i} f \Delta A_3 + \alpha A_3 = -Q A_2, \tag{2}$$

$$\partial_z A_2 - \mathrm{i} f \Delta A_2 + \alpha A_2 = \bar{Q} A_3, \tag{3}$$

$$\partial_z A_4 + \mathrm{i} f \Delta A_4 - \alpha A_4 = -\bar{Q} A_1, \tag{4}$$

where Δ is the transverse Laplacian $\partial_x^2 + \partial_y^2$ and f is a parameter controlling the diffraction. In scaled coordinates it is proportional to the inverse of the Fresnel number $f = (4\pi F)^{-1}$. The Fresnel number for this geometry is given by $F = n_0\sigma^2/\lambda d$, where n_0 is the background index of diffraction, σ is the full-width half maximum of a incoming pump beam, λ is its wavelength and d is the length of the PR crystal (propagation length). The bar denotes complex conjugation, α is the linear absorption and Q is the amplitude of the grating in the crystal that is generated by the interference of waves at allowed k-vectors. In the operation of DPCM only transmission gratings are allowed. The temporal evolution of Q is well approximated by a relaxation equation of the form:

$$\tau\partial_t Q + Q = \frac{\Gamma}{I}(A_1\bar{A}_4 + \bar{A}_2 A_3), \tag{5}$$

where τ is the relaxation time of the grating, I is the total intensity and Γ is the PR coupling constant times the crystal thickness. We are not concerned with the frequency detuning, and are assuming diffusion dominated build-up of gratings. Hence Γ is real. The waves are following the changes in the crystal adiabatically, therefore the temporal derivatives in Eqs. (1)–(4) are ignored. Likewise, the spatial derivatives are neglected in Eq. (5), since the diffusion effects are controlled by the slow electronic processes in PR crystals. For the moment we consider only one transverse dimension (x).

The stationary absorptionless plane-wave solutions ($\Delta = \partial_t = 0$) of Eqs. (1)–(4) and (5) are of the form [6]:

$$A_1 = C_4\sin(\theta - \theta_0), \quad A_2 = C_2\cos(\theta_d - \theta), \tag{6}$$

$$A_3 = C_2\sin(\theta_d - \theta), \quad A_4 = C_4\cos(\theta - \theta_0), \tag{7}$$

where

$$\tan(\theta) = \tan(\theta_0)\exp(a\Gamma z), \tag{8}$$

and the parameter a is found from the transcendental equation

$$a = \tanh(a\Gamma/2). \tag{9}$$

A sharp threshold oscillation condition $\Gamma_{\text{th}} = 2$ follows from this equation. The boundary values θ_0 at $z = 0$ and θ_d at $z = d$ are given by:

$$\tan(\theta_0) = \exp\left(-\frac{a\Gamma}{2}\right)\sqrt{\frac{a - q^*}{a + q^*}},$$
$$\tan(\theta_d) = \exp\left(\frac{a\Gamma}{2}\right)\sqrt{\frac{a - q^*}{a + q^*}}, \tag{10}$$

where $q^* = (|C_4|^2 - |C_2|^2)/(|C_4|^2 + |C_2|^2)$ is the ratio of the input power flux to the total input intensity. Here $C_2 = A_2(z = d)$ and $C_4 = A_4(z = 0)$ denote the given initial amplitudes. From these equations one obtains simple relations between the experimentally measured quantities, the transmissivities $T_0 = |A_{1d}/C_4|^2$, $T_d = |A_{30}/C_2|^2$ and the reflectivities $R_0 = |A_{30}/C_4|^2$, $R_d = |A_{1d}/C_2|^2$ at the opposite faces of the crystal:

$$T_0 = T_d = T = \sin^2(u), \quad R_0 = T/q, \quad R_d = qT, \tag{11}$$

where $u = \theta_d - \theta_0$ is the so-called total grating action, and $q = |C_4/C_2|^2$ is the ratio of input beam intensities. We use these solutions and relations as a check on our numerical procedure for the treatment of Eqs. (1)–(5).

The procedure for solution of Eqs. (1)–(5) with the inclusion of transverse effects consists of a beam propagation method [7] for the spatial problem of $A_1 - A_4$, and a Runge-Kutta-like method for the temporal problem of Q. The details of the method are given elsewhere [8]. In the plane-wave case ($f = 0$) we find excellent agreement between the numerical and the analytical solution (see Fig. 2a). If transverse effects are included and Gaussian input beams are used, the intensities of the PC beams (I_1, I_3) decrease (Fig. 2b) as compared to the plane-wave case. The inclusion of diffraction is always detrimental to the process of OPC. Likewise, the inclusion of absorption is also detrimental to the process of OPC.

Further, with the inclusion of transverse effects an interesting phenomenon is observed: self-bending of PC beams. Fig. 3a depicts a tilt of the beam I_3 away from the center at $x = 0$. This is a genuine transverse effect, caused (and controlled) by the finite beam waist and not the (possible) phase mismatch. The

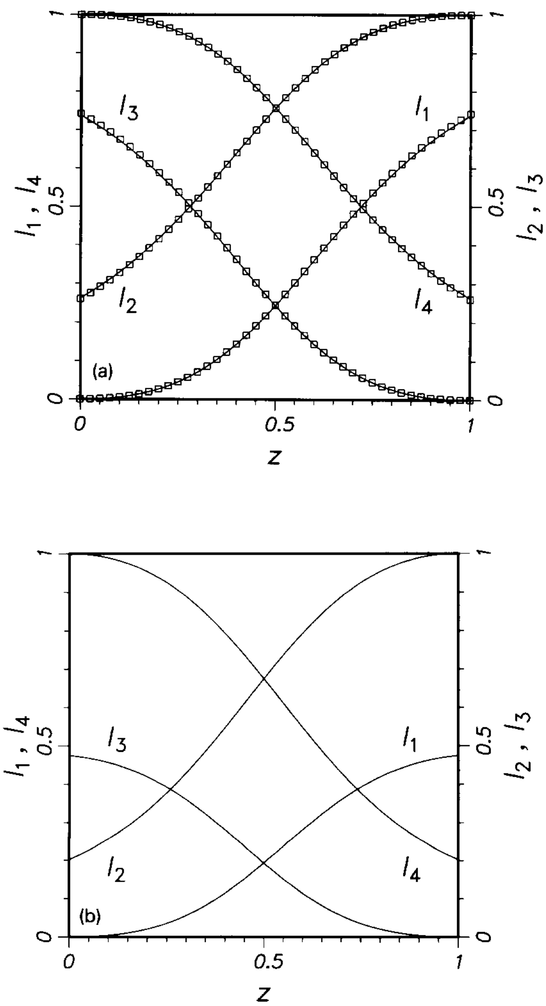

Fig. 2. (a) Checking numerics with the plane-wave solutions (points: analytical results, curves: numerical results). (b) Intensities $I_i(x = 0, z)$ in the crystal with Gaussian input beams $I_{2,4} = C_{2,4} \exp(-x^2/\sigma^2)$ and finite f ($f = 0.01, \Gamma = 3, C_{2,4} = 1, \sigma = 0.2$).

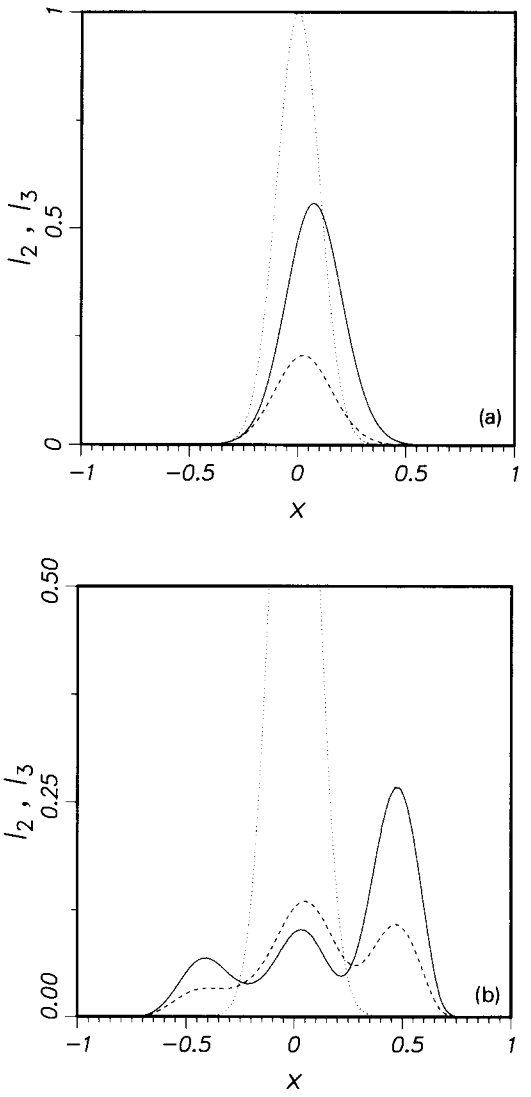

Fig. 3. Output profiles $I_2(x)$ (dashed line) and $I_3(x)$ (full line) versus the transverse coordinate x at $z = 0$. (a) $f = 0.01$, (b) $f = 0.05$. For reference, the pump beam $I_4(x)$ (dotted line) is also plotted. In (a) self-bending of beams is evident, in (b) they acquire a more complicated transverse structure. Other parameters are as in Fig. 2.

physical origin of self-bending is the convective flow of energy out of the interaction region [5]. It leads to the degradation of PC beam quality. Self-bending is observed not only in the double phase conjugation, but in the standard OPC as well.

If the influence of diffraction is increased (higher value of f), a further spatial widening of the PC beam profile is observed, with the appearance of two (or more) local maxima of $I_3(x)$, due to self-defocusing in the medium (Fig. 3b). The drift of output profiles, as mentioned, is caused by the convective flow of energy out of the interaction region. This is confirmed if one plots the amplitude of the grating Q in the crystal (Fig. 4a). The amplitude acquires an asymmetric spatial distribution. The self-bending and self-defocusing of A_3 can clearly be seen in the transverse beam distribution, as it propagates through the crystal (Fig. 4b).

Our results agree with the recent work on two-dimensional DPCM [5] only for small influence of diffraction and only qualitatively. We confirm that in

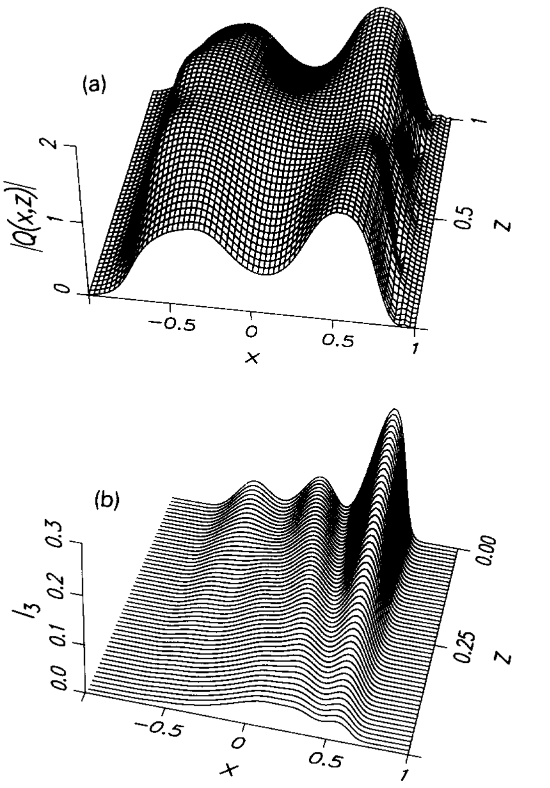

Fig. 4. Intensity of (a) the grating amplitude Q and (b) the PC beam I_3 in the crystal ($\Gamma = 3, \sigma = 0.2, f = 0.05$).

this case DPCM is not a simple oscillator. The most appropriate description is that it represents a convective oscillator. Transverse convection is evident in the system as soon as the transverse spread of beams is taken into account. Also, the existence of a diffuse threshold for oscillation is evident in our numerical simulations [8]. The sharp threshold condition on Γ from the plane-wave case now broadens out. Different seeds are amplified to different levels. However, with increasing Γ and f the DPCM becomes a poor (nonlinear and unstable) amplifier. An instability threshold is soon reached, where temporal behavior becomes more complicated. Strong couplings and strong diffraction lead to more complicated spatio-temporal phenomena. The reason for discrepancy with Ref. [5] is probably that the theory advanced there is linear (it applies to low reflectivity levels) and that it is derived for a rather special two-dimensional geometry. It contains no transverse Laplacian.

The inclusion of transverse effects improves the agreement between theoretical and experimental results. The theory based on plane-wave analysis consistently gives too high estimates for the intensity reflectivity. Also, it can not account for the asymmetry observed experimentally in the plots of transmissivities and reflectivities as functions of the input beam ratio. However, when transverse effects are included the numbers are comparable. This remains true in both one or two dimensional transverse case (however, the value of f might be different in the two cases). Fig. 5 offers a comparison between the numerical results with transverse effects and the experimental results. Note the asymmetry in the experimental curves for the transmissivities and the corresponding asymmetry in the numerical curves. It should also be noted that f is used here as a parameter displaying the correcting influence of transverse effects, and not as an experimental datum. The experimental value of f is not known to us. An even better agreement with experiment is obtained by including both the transverse effects and the linear absorption. The curves can also be fit by the absorption only, however the asymmetry is missing then.

As mentioned, increasing Γ beyond the oscillation threshold leads to complicated oscillations. In Fig. 6 we present oscillations (limit cycles) in the reflectivity and the transmissivity on both faces of the crystal. Now the transmissivities T_0 and T_d are different from

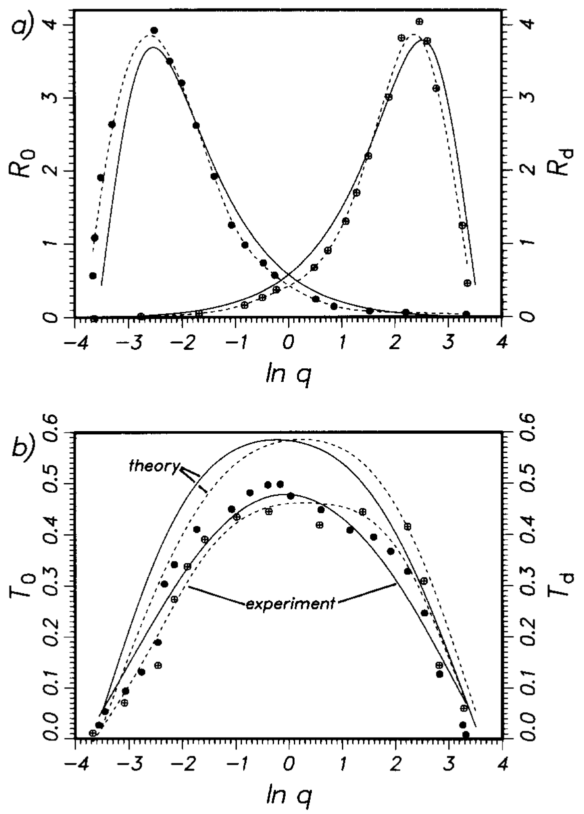

Fig. 5. Comparison with the experimental results of Ref. [3]. (a) Reflectivities R_0, R_d at both sides of the crystal as functions of the pump ratio q. Filled dots are the experimental values of R_0, crossed dots are the values of R_d. Dashed lines are polynomial fits through the experimental points. Full lines are numerical curves with $\Gamma = 4$ (as suggested in [3]) and $f = 0.121$. (b) Corresponding transmissivities T_0, T_d. One of the full lines is a fourth-order polynomial fit through the experimental points for T_0 (to guide the eye), and the other is the corresponding numerical curve. Dashed lines are the same for T_d. Numerical curves are kept separated from the experimental ones, in order to display the symmetry breaking.

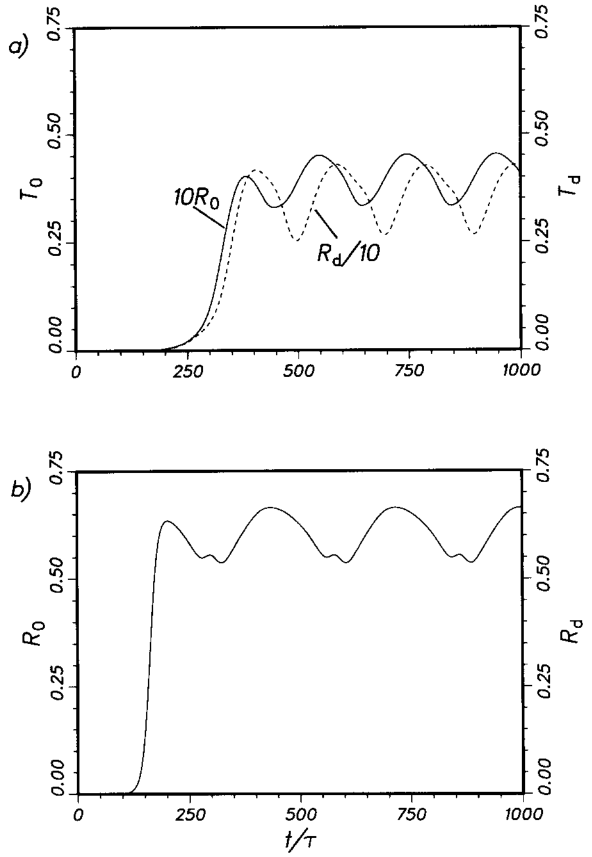

Fig. 6. Limit cycle oscillations of the transmissivities and the reflectivities for two values of the pump ratio q. Here, as in Fig. 5, the transverse coordinate is integrated out. a) Transmissivities T_0 (solid line) and T_d (dashed line) at $q = 10$. The same curves (on a different scale) also depict R_0 and R_d. b) Reflectivities R_0 and R_d at the degenerate point $q = 1$. At degeneracy all four quantities are the same, and execute the same motion.

each other, however the formulae $R_0 = T_0/q, R_d = qT_d$ from the plane-wave case still hold. These oscillations become more irregular as the value of the wave coupling constant Γ is increased. However, no chaotisation is observed, unless Γ is made complex [8,9].

The inclusion of the other spatial dimension (y) leads to rich spatial and temporal phenomena. We arrange the axes in the $x-y$ plane so that the wave vector of the grating points along the $y = x$ direction. Here we confine ourselves only to the approach to stable transverse patterns. Fig. 7 depicts a stable two-dimensional pattern that has developed in the DPCM with two transverse dimensions, starting from simple Gaussian humps for all beams. The primary peaks arrange themselves along the $y = x$ axis, while the secondary peaks appear to the left and right of the axis, keeping the order and the prescribed $x \leftrightarrow y$ symmetry in force. This general arrangement is consistent with the latest experimental results [10]. Depending on the initial and the boundary conditions, other stable patterns are possible. Again, increasing Γ leads to complicated dynamics of patterns. However, no chaos is observed as long as Γ remains real. An analysis of

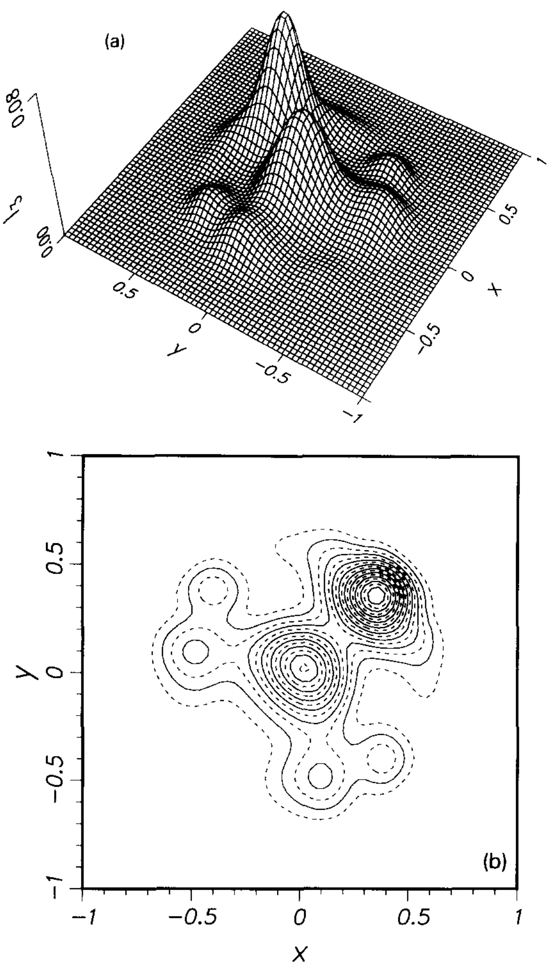

Fig. 7. Stable two-dimensional pattern of I_3 at $z = 0$ after 1000 temporal cycles, with $\Gamma = 3, f = 0.05$. b) Corresponding contour-plot displaying more clearly the built-in $x \leftrightarrow y$ symmetry of equations. This symmetry is highlighted by the chosen $y = x$ direction of the gratings wave vector.

stable patterns that can develop in PR oscillators is the first step in an investigation of spatio-temporal instabilities and chaotization through the generation and dynamics of structural defects. Our current research efforts are directed toward the analysis of two-dimensional patterns and dynamical effects that follow their formation [8].

In summary, we have considered transverse and temporal effects in DPCM. We find that the inclusion of the finite lateral beam extension lowers the reflectivities, bends the beams, speeds-up the convergence and resolves the controversy about the nature of double phase conjugation.

In the plane-wave approximation the DPCM is an oscillator with a gain threshold. Exponential growth of PC beams is observed above the threshold for arbitrarily small seeds. Nothing is observed below the threshold.

In the transverse case the DPCM is a convective oscillator. The gain threshold is not well defined. It is smeared over an interval. Different seeds are amplified to different levels. Intensity distributions of PC beams are shifted and asymmetric. The bending of beams is caused by the convective flow of energy. The intensity reflectivity attained is lower than in the plane-wave case. However, the transverse case converges more rapidly. For strong couplings and strong diffraction, the DPCM becomes a poor phase conjugator.

The inclusion of transverse effects improves the agreement with the experimental results. It helps explain the experimentally noted asymmetry in the transmissivities at the opposite faces of the crystal as a function of the input beam ratio. Asymmetry is the consequence of the nonreciprocity in the process of double phase conjugation, and is absent in the plane-wave theory. Finally, for strong couplings instabilities are observed, in that the reflectivities do not settle onto any one value, but oscillate regularly or irregularly (for Γ complex). Such spatio-temporal instabilities are under current investigation.

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