Multigrating optical phase conjugation: numerical results

Milivoj. R. Belić* and Wiesław Królikowski†

Max-Planck Institut für Quantenoptik, D-8046 Garching, Federal Republic of Germany

Received October 11, 1988; accepted January 23, 1989

Multigrating optical phase conjugation in photorefractive media in the case of pump depletion and for arbitrary relationships among different coupling strengths is studied numerically. The role of linear absorption as well as the role of all the coupling mechanisms and their competition in the wave mixing process is examined. It is found that multigrating operation with accidental values of coupling constants leads to a decrease in the efficiency of the process. It is also shown that the presence of absorption, which is in principle detrimental, sometimes exerts a beneficial influence on multigrating phase conjugation.

The field of optical phase conjugation (OPC) has been the subject of deep exploration during the past decade. Phase conjugation offers the possibility (and reality!) of a great many practical applications in image processing, photolithography, holographic interferometry, aberration correction, etc.[1] One of the powerful techniques used to generate a phase-conjugate wave is four-wave mixing (4WM) in photorefractive media.[2] Four light beams pass through the photorefractive crystal in this process. The interference of these beams creates a complex interference pattern, which leads to the modulation of the refractive index through the photorefractive effect. In this way the index grating is written in the crystal. This refractive-index grating acts as a thick phase hologram and causes beams to diffract and exchange energy.

The theoretical description of this phenomenon is based on coupled-wave equations that describe wave interaction as nonlinear coupling through the index grating. This phenomenon has been studied by many researchers. Initially, simple linearized models of interaction, such as the undepleted-pumps approximation, were considered.[3] Next the more realistic models, including pump depletion[4] and linear absorption,[5] were successfully treated. A common feature of all these approaches is that they include only one coupling mechanism (one grating). However, owing to the nature of the 4WM process in photorefractive crystals, there may be, in general, four different index-grating coupling mechanisms that may contribute to the process.[6] The main argument in favor of the one-grating approximation is that in experimental conditions it is always possible to eliminate all undesired couplings and leave only one main coupling responsible for phase conjugation. Furthermore, some experimental and theoretical[6,7] results suggest that in some situations it is advantageous to operate in the single-grating regime. Appropriate crystallographic orientation, choice of beam polarization, etc. cause only one grating to be recorded effectively.

There is, however, a holographic arrangement of four-wave interaction in photorefractive crystals in which it is practically impossible to avoid the presence of many coupling mechanisms in the process. This is the so-called self-pumped phase-conjugate mirror proposed by Feinberg.[8] In this geometry the electro-optic crystal exhibiting a strong photorefractive effect is illuminated by only one laser beam. Because of the fanning effect[9] and total internal reflection on the crystal faces, the pump beams are created. Since all interacting beams are coherent and have the same polarization, they can write different gratings inside the crystal. One should expect some kind of competition among different gratings. Indeed, it was shown recently that such a multigrating regime may lead to temporal instabilities in the output phase-conjugate intensity.[10] Therefore it seems to be necessary from the practical point of view to investigate the properties of optical phase conjugation through multiple gratings.

The first accounts dealing with that problem were concerned primarily with specific cases of intensity and coupling strength relationships, namely, with the strong-pump limit[11] and with equal coupling strengths for transmission and reflection gratings.[7] In these cases it was even possible to obtain some exact results. Some interesting features of OPC under these conditions have been revealed. It has been shown, for instance, that the phase-conjugate reflectivity exhibits saturation when transmission and reflection gratings act simultaneously with large and equal coupling strength.

The purpose of this paper is to study multigrating phase conjugation through four-wave mixing in photorefractive crystals when pump depletion, linear absorption, and arbitrary relationships among coupling strengths of different gratings are all allowed. We shall show the role of each coupling mechanism and the competition among these mechanisms in the phase-conjugation process.

The standard geometry of 4WM is considered. The slab of a photorefractive crystal, situated between the planes $z = 0$ and $z = d$, is illuminated by two counterpropagating pump waves (A_1 and A_2) and by the signal wave (A_4). As the result of nonlinear interaction inside the medium, a phase-conjugate replica (A_3) of the signal wave is created. This wave propagates opposite A_4. Assuming that each beam has the form of a plane wave, the interaction among them in the slowly varying envelope approximation is described by the following set of coupled equations:

$$I_0 A_1' = g_T A_T A_4 - g_R A_R A_3 - g_P |A_2|^2 A_1 - \alpha I_0 A_1, \tag{1a}$$

$$I_0 A_2^{*\prime} = g_T A_T A_3^* - g_R A_R A_4^* - g_P A_2^* |A_1|^2 + \alpha I_0 A_2^*, \tag{1b}$$

0740-3224/89/050901-09$02.00

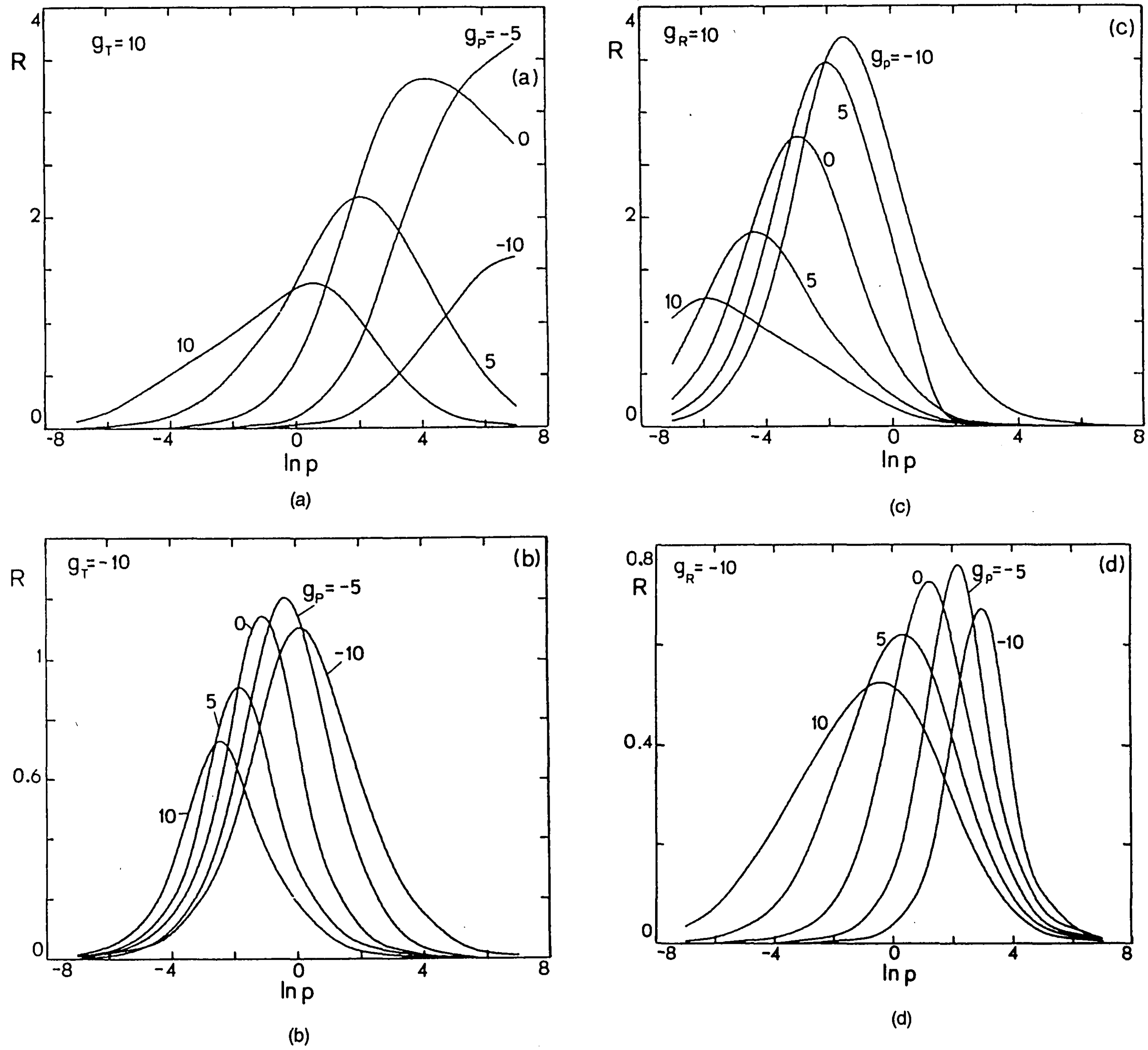


Fig. 1. Effect of pump coupling on OPC through (a), (b) the transmission and (c), (d) the reflection grating.

$$I_0A_3' = -g_TA_TA_2 - g_R{}^*A_R{}^*A_1 - g_S|A_4|^2A_3 + \alpha I_0A_3, \quad \text{(1c)}$$

$$I_0A_4{}^{*\prime} = -g_TA_TA_1{}^* - g_R{}^*A_R{}^*A_2{}^* - g_S|A_3|^2A_4{}^* - \alpha I_0A_4{}^*, \quad \text{(1d)}$$

where the primes denote differentiation in the propagation z direction and the stars denote the complex conjugate. I_0 is the total light intensity, $I_0 = \sum|A_j|^2$, and α is the absorption coefficient. Different coupling constants g are related to material properties of the crystal and are of the form $g = in \exp(i\phi)$, where ϕ is the spatial phase shift between the index grating and the interference pattern.[2] Interference terms $A_T = A_1A_4{}^* + A_2{}^*A_3$ and $A_R = A_1A_3{}^* + A_4A_2{}^*$ are connected with the transmission and the reflection gratings, respectively. These two gratings are responsible for the generation of the phase-conjugate wave (PCW). Besides these, there are also two contributions, namely, $A_4A_3{}^*$ and $A_1A_2{}^*$, coming from two-wave mixing. They arise from the signal–phase-conjugate and the pump–pump interaction. It should be pointed out that, in general, the value of the photorefractive phase shift ϕ depends on the material and geometric factors. However, in most practical situations and in many photorefractive crystals this phase shift equals $\pi/2$. In other cases it is possible to obtain the $\pi/2$ shift by means of recently developed techniques, such as recording running holograms[12] and using an ac external electric field.[13] As is well known, such a shift is of great practical interest, since it permits the largest energy transfer between interacting beams.[14] In what follows we restrict ourselves to that case and assume that the phase shifts connected with different gratings are the same and equal exactly $\pi/2$.

Our task is to solve coupled Eqs. (1) with split boundary conditions specified on two opposite crystal faces: $A_1(z=0) = A_{10}$, $A_4(z=0) = A_{40}$ and $A_2(z=d) = A_{2d}$, $A_3(z=d) = 0$. We shall do this numerically, using the shooting method. This procedure was recently successfully applied to study different configurations of wave mixing.[15]

Results of our numerical calculations are presented in Figs. 1–9. As the basic output quantity we chose the phase-conjugate reflectivity (PCR), which is the ratio of the output intensity of the PCW to the input signal intensity, $R = I_3(0)/I_{40}$. This quantity enables one to treat the 4WM scheme as a phase-conjugate mirror with given reflectivity R. We plot the PCR as a function of the input pump ratio $p = I_{2d}/I_{10}$. Various parameters include coupling constants (given in units of inverse centimeters), the absorption coefficient, and the input signal intensity. The last quantity is assumed to be 0.1 (normalized to the total pump intensity) in most of the calculations (except in Fig. 9, where it is varied). The crystal thickness is set to $d = 0.3$ cm throughout the calculations.

First we show the influence of the coupling between counterpropagating waves on the generation of the PCW through the transmission and reflection gratings. Figure 1 displays the effect of pump coupling on the process. It is seen that this direct pump interaction strongly affects the PCR. First, it produces a strong horizontal shift of the plots. This effect is caused by the energy transfer between pumps. Energy exchange between the pumps is in some sense equivalent to the change in boundary conditions for them. The signs of the coupling constant, g_P, decides the direction of the energy transfer and of course the plot shift. The second aspect of the pump coupling is the change of the maximal

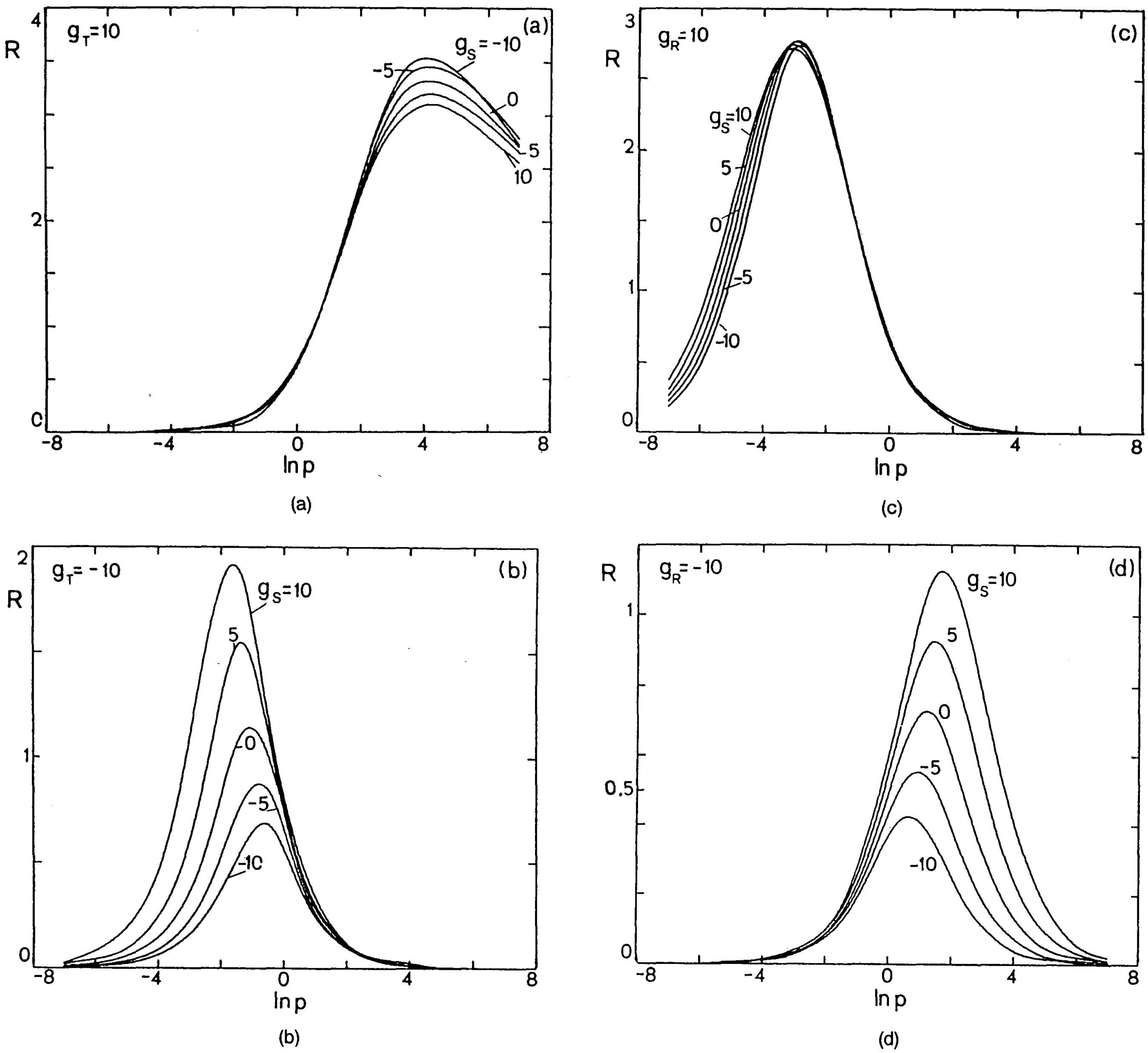


Fig. 2. Effect of signal–phase-conjugate coupling on OPC caused by (a), (b) the transmission and (c), (d) the reflection grating.

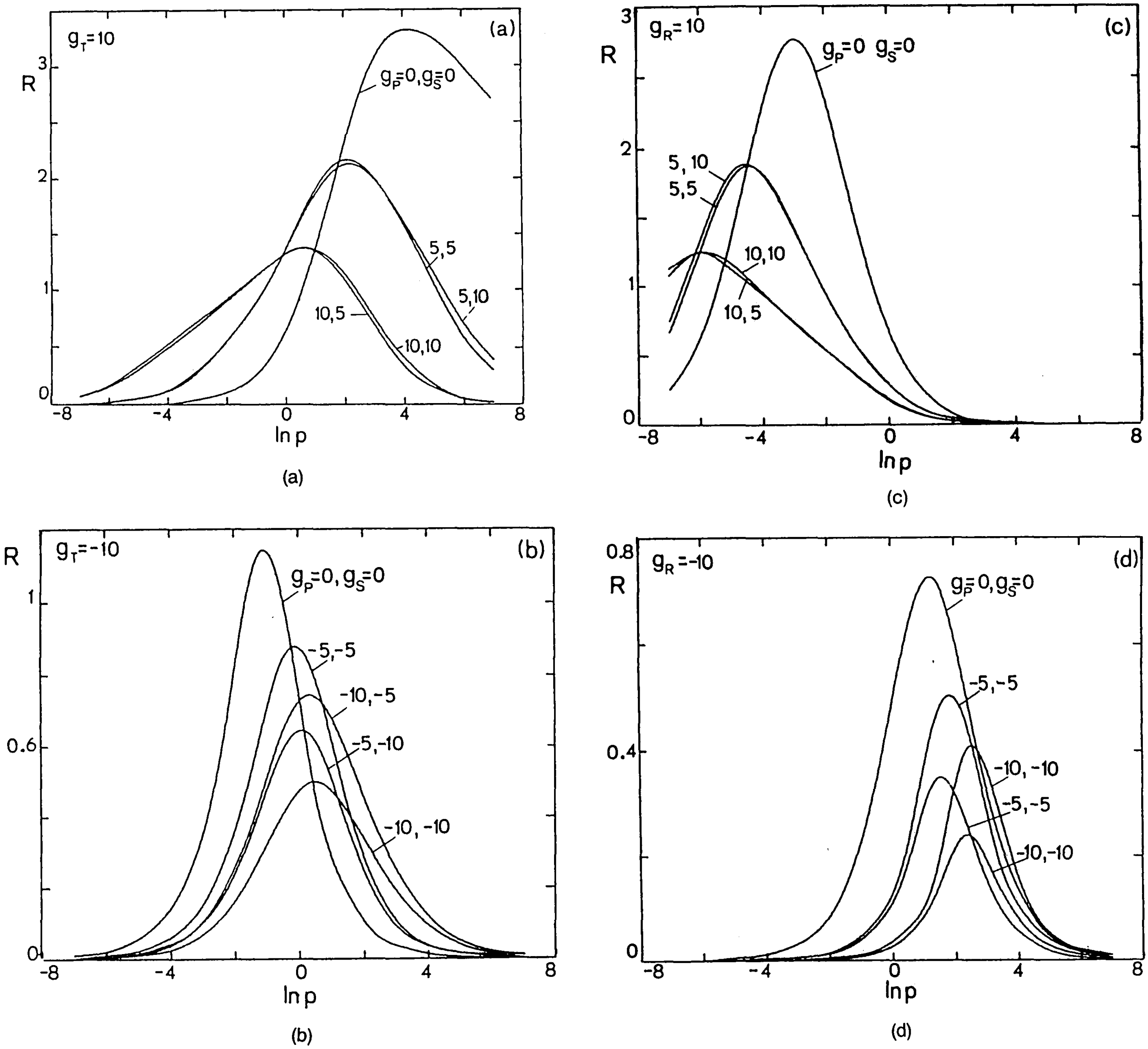


Fig. 3. Simultaneous coupling between counterpropagating waves in OPC through (a), (b) the transmission and (c), (d) the reflection grating.

value of the PCR. It is seen from Fig. 1 that for negative g_P this extra interaction improves efficiency of the phase conjugation, giving a higher PCR. This property is especially useful in the case when the reflection grating is responsible for phase conjugation. Then the grating period is small, and the saturation effect during the hologram recording is noticeable. It leads to a drastic decrease of the mixing efficiency.[16]

In Fig. 2 the role of the second additional two-wave coupling is displayed. It involves the coupling between the signal wave and its phase-conjugate replica. In the strong-pump theory of multigrating OPC this coupling mechanism was not taken into account.[11] It is seen now that the shifts of the plots are not so strong as previously. This mechanism does not lead to direct energy exchange between pumps. Furthermore, this coupling affects the phase conjugation appreciably only in the case of a negative value of the main coupling constant g_T (or g_R). The positive coupling constant, g_S, then leads to an appreciable increase in the PCR. For positive g_T (or g_R) the influence of this mechanism is weak, independently of the sign of g_S.

Figure 3 illustrates the simultaneous presence of both two-wave couplings in OPC. Note that an increase in the efficiency of this process occurs when the coupling constants g_P and g_S satisfy the same conditions as in the situation when each of these mechanisms individually modifies the 4WM.

The subsequent multigrating arrangement of the phase-conjugation process is photorefractive crystals involves the simultaneous existence of the two main coupling mechanisms responsible for the creation of a PCW, namely, both the transmission and the reflection gratings. The most characteristic examples of the competition between these two mechanisms are shown in Figs. 4 and 5. In each case the coupling strength of one grating is kept fixed (transmission in Fig. 4 and reflection in Fig. 5) while the coupling strength of the other is changing. In this manner one can study the influence of the reflection grating on OPC realized by the transmission grating, and vice versa. It is clearly seen from the figures that the presence of an additional phase-conjugate mechanism in some cases improves the efficiency of the process. The improvement depends, however, not only on the relation between coupling strengths for a particular grating but also on their absolute values. So, for small values, and with the same sign of g_T and g_R, the cooperation of two gratings is constructive, leading to a higher PCR in comparison with that obtained in the single-grating operation [Figs. 4(a) and 5(a)]. On the other hand, when the coupling strength for the main grating is high, there is a narrow region of the values for the second g that give the increasing PCR, and the signs of the g's should be opposite. From Figs. 4(b) and 5(b) it is also evident that the reflectivity R drastically decreases when the coupling strengths for both gratings become comparable in magnitude and have opposite sign. One can show that, when $g_T = -g_R$, a PCW is not generated.[7] The reason is that the contributions to the PCW coming from the diffraction of the pumps on different gratings are out of phase and interfere destructively. Since their magnitudes are the same, there is no output PCW.

On the other hand, the multigrating OPC with equal coupling strengths ($g_T = g_R = g$) leads to the saturation of the PCR below unity. This effect, which is also seen in Figs. 4 and 5, has been discussed in Ref. 7. It followed there as a result of numerical calculations. Here we show that the limitation of the PCR may easily be drawn from coupled equations, without solving them. From Eqs. (1c) and (1d) one can get equations for the intensities of the interacting waves and also for the difference $I_4 - I_3$. These equations may be formally integrated, and for real g ($\pi/2$ photorefractive phase shift) one obtains

$$(I_4 - I_3) = (I_4 - I_3)_{z=d} \exp\left[2g \int_d^z (I_1 + I_2)/I_0 dz'\right]. \qquad (2)$$

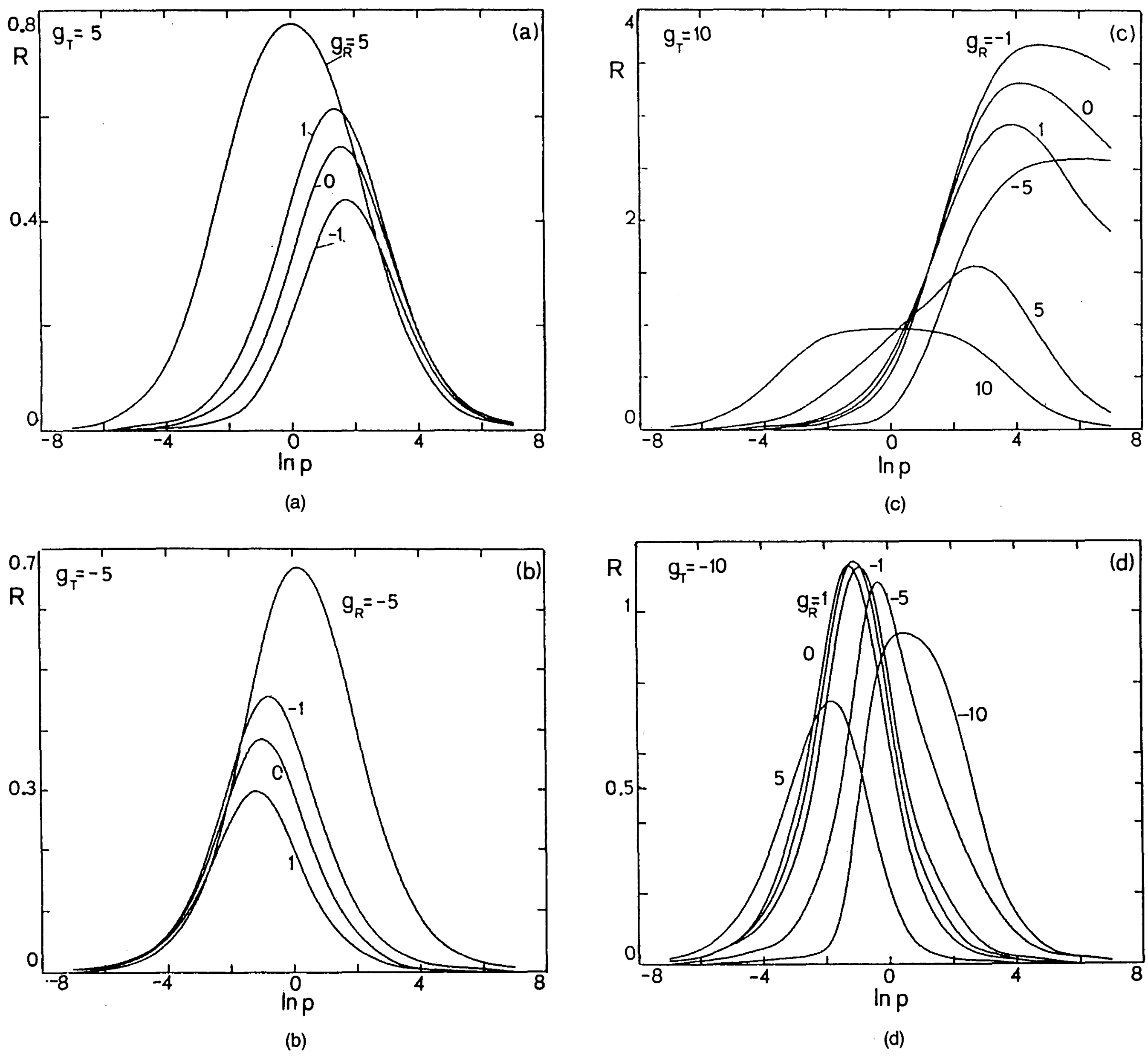
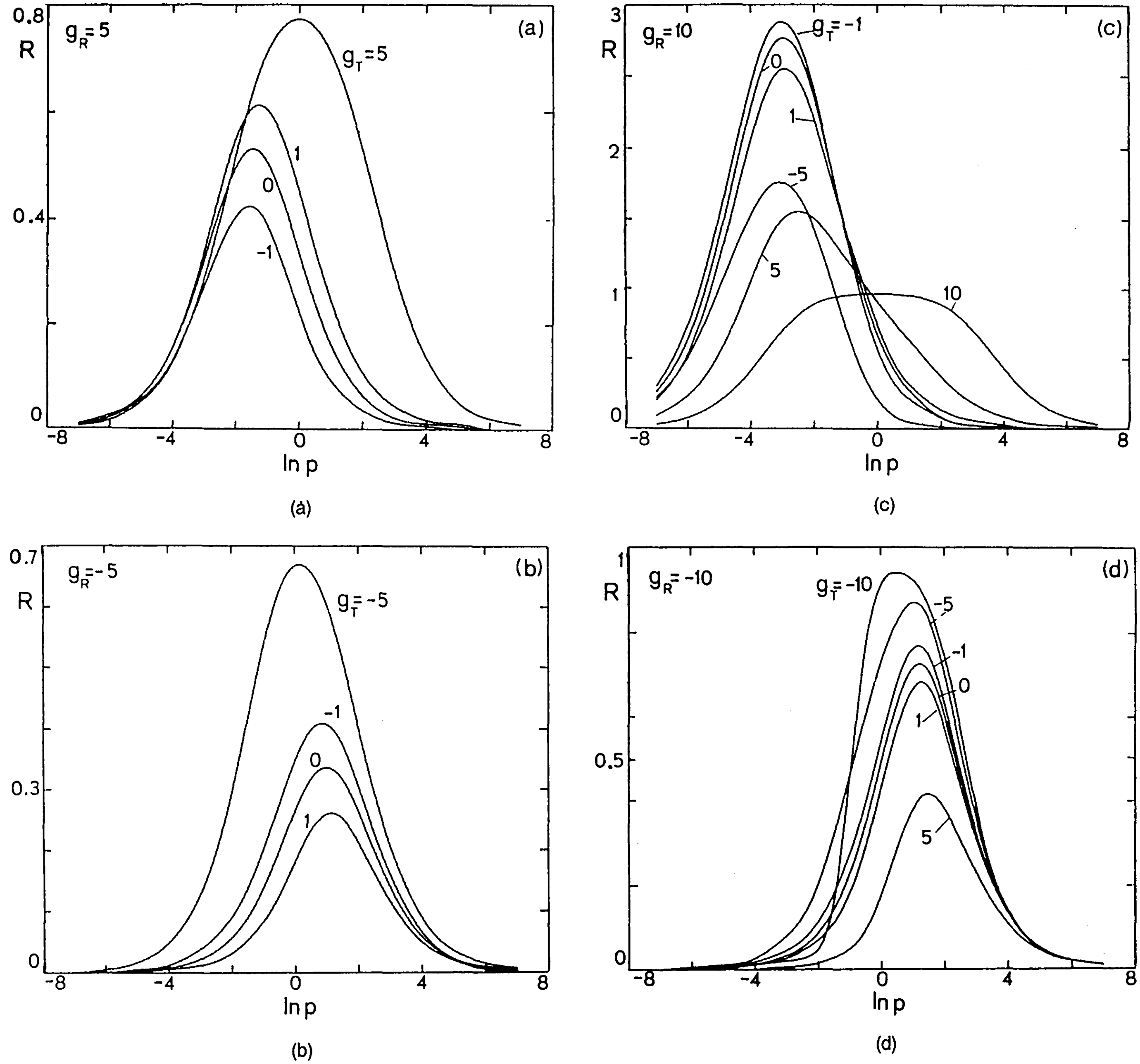


Fig. 4. Effect of the reflection grating on the OPC produced by the transmission grating.



(a) (c)

(b) (d)

Fig. 5. Converse of Fig. 4: effect of the transmission grating on phase conjugation produced by the reflection grating.

Since in the plane $z = d$ the phase conjugate PCW does not exist $[I_3(d) = 0]$ and since $I_4 > 0$, it follows from Eq. (2) that $I_4 > I_3$ everywhere, and of course $R < 1$, independently of the values of the other coupling constants g_P and g_S. On physical grounds this effect may also be explained as follows. Let us note first that in the single-grating operation each pump beam plays a specific role. In the transmission geometry the pump A_1 is the so-called writing beam, and A_2 is the reading beam. Diffraction of the latter on the hologram written by the former builds the PCW. In the reflection geometry the roles of the pumps are reversed. Now A_2 is the writing beam and A_1 reads off the hologram. As is well known, in single-grating operation the optimal (maximal) value of the PCR is connected with the strong asymmetry in the pumping. In the case under consideration, i.e., for $g_T = g_R$, there is no such difference between pumps as far as their participation in the appearance of the PCW is concerned. Now phase conjugation takes place as if only one grating existed with the coupling constant twice that of g (stronger g dependence) but with symmetric pumping. This last feature leads to the limitation of PCR. The larger gd is, the stronger the depletion of the signal beam becomes, and the saturation of PCR takes place. Therefore, for large coupling strengths, the single-grating operation is more effective.

On the other hand, for small gd, stronger g dependence leads to higher values of PCR in comparison with the single-grating case. The critical value of gd above which the equal-strength multigrating OPC is ineffective is of the order of unity. In the weak-signal limit it is possible to derive this quantity exactly, and it equals 1.6.[11] Now it becomes clear why, for $g_T = 5$ or $g_R = 5$ (Figs. 4 and 5) every contribution of g_R (g_T) of the same sign improves R. The generation of the PCW then takes place in conditions far from saturation, and therefore the value of R is higher than the one obtained in the single-grating regime with $g_T = 5$ or $g_R = 5$. Generally speaking, all cases of given g_T and g_R fall between two limiting cases: saturation of PCR when $g_T = g_R = g$ and no PCW generation when $g_T = -g_R$. For a large value of g_T (g_R), only a weak contribution of g_R (g_T), usually of the opposite sign, improves the OPC process. Any larger contribution of the other coupling responsible for the generation of a PCW brings us nearer one of the limiting states, and this happens independently of the values of coupling strengths for two-wave interactions.

With respect to full multigrating phase conjugation, i.e., with all coupling mechanisms turned on, it is difficult to discuss that case because many possible relations among different coupling strengths exist. However, numerical calculation can still show the individual effect of any particular mechanism of interaction on the phase conjugation. We present some examples of such operation in Fig. 6. In general it is difficult if not impossible to give precise conditions that all couplings should satisfy if one is to obtain the most effective phase conjugation. An accidental set of different values of the coupling parameters usually leads to a decrease in the maximal value of PCR in comparison with the single-grating case.

In the remainder of this paper we discuss the influence of two other parameters affecting multigrating OPC: absorption and the intensity of the input signal. The role of absorption is especially interesting. It has two different aspects. The first one is destructive. The larger the absorption coefficient, the smaller the output phase-conjugate intensity and PCR. This behavior is as one would expect. The second aspect is more interesting, in that absorption can modify some properties of the multigrating interaction in a more complicated fashion. We illustrate this in Fig. 7, in which the influence on the PCR of the coupling between the signal and its conjugate is presented for different absorptions. In the case of no absorption this additional interaction decreases the efficiency of the process. However, when

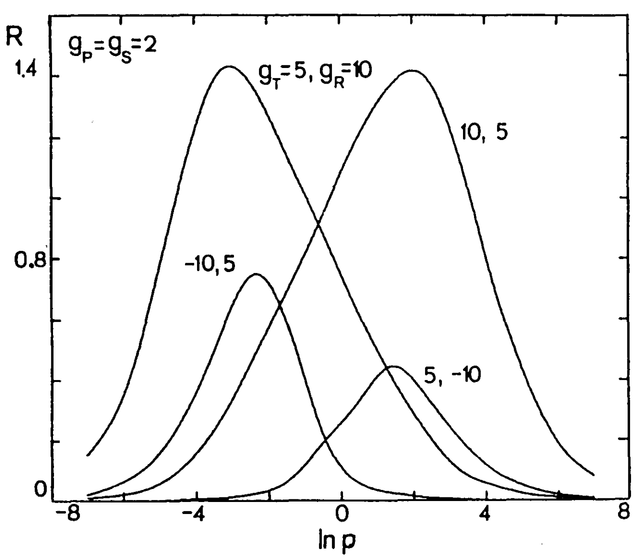


Fig. 6. Full multigrating phase conjugation.

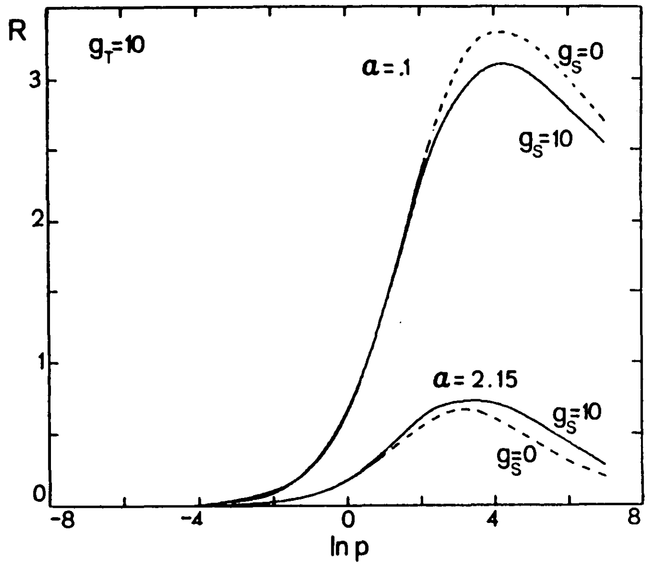


Fig. 7. Influence of absorption on multigrating phase conjugation.

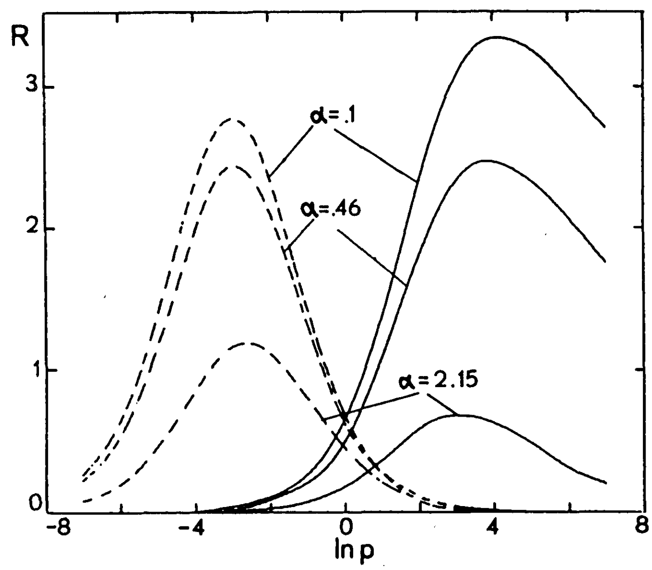
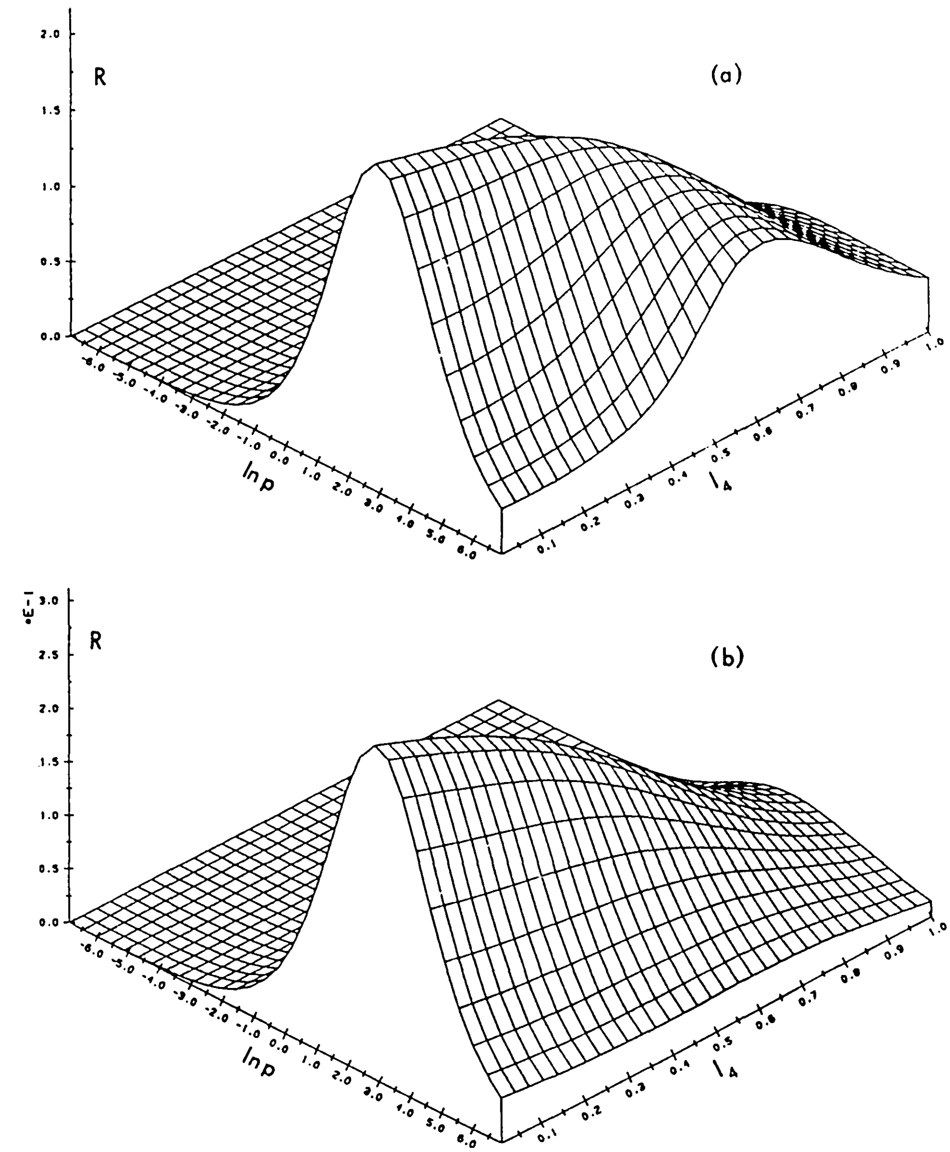


Fig. 8. Role of absorption in single-grating phase conjugation: solid curves, transmission grating; dashed curves, reflection grating.

the absorption of light is included, the coupling between the signal and the conjugate wave is constructive and gives higher PCR. This behavior seems to follow from the selective absorption influence on various coupling mechanisms. The effect is the subject of current investigation. We observed it in our numerical studies. A distinct feature of this influence is that it is not unique and depends on the values and signs of the coupling strengths in a complicated manner. Therefore in Fig. 8 we give only an example of this effect, for the case when OPC occurs through the transmission and/or the reflection grating.

To complete our studies of multigrating phase conjugation, we illustrate the role of the input signal's intensity in this process. In the series of three-dimensional graphs (Fig. 9) we plot the PCR as a function of the pump ratio p and the signal ratio $q = I_{40}/(I_{10} + I_{2d})$. The parameters are the coupling strengths. In all plots a value of $\alpha = 1\ \mathrm{cm}^{-1}$ for the



(figure continued)

absorption coefficient is assumed [except in Fig. 9(b), where $\alpha = 4.15\ \mathrm{cm}^{-1}$]. The coupling strengths are chosen to refer to some previously discussed cases of multigrating operation. Thus in Figs. 9(c) and 9(d) the effect of the pump coupling on OPC is displayed. It is seen that, in general, increasing the input signal intensity leads to a decrease of PCR. However, sometimes the behavior of PCR versus signal intensity is slightly different. One such case was presented in a previous paper (Fig. 5 of Ref. 7). The PCR is almost constant in the broad range of signal ratio values when the coupling strengths for transmission and reflection gratings are the same. The phase conjugator acts as a real phase-conjugate mirror, and then the output amplitude of the conjugate wave is almost independent of the impinging signal wave.

In conclusion, phase conjugation in photorefractive crystals with more than one active coupling mechanism has been studied numerically. The roles of various gratings and their competition in the process have been investigated. It has been shown that in general the multigrating operation with accidental values of coupling parameters leads to a decrease in the efficiency of the process. However, a careful choice of these quantities may improve generation of PCW's. This may happen when the pump coupling with the negative sign of the coupling constant is allowed in the 4WM process. It has been also shown that the presence of absorption, which is in principle detrimental, may sometimes change the properties of competition among different gratings.

ACKNOWLEDGMENTS

M. R. Belić thanks the Humboldt Foundation for financial assistance during his stay in the Federal Republic of Germany. The authors thank H. Walther for his hospitality at the Max-Planck Institut für Quantenoptik.

* Permanent address, Institute of Physics, P.O. Box 57, 11001 Belgrade, Yugoslavia.

† Permanent address, Institute of Physics, Polish Academy of Sciences, 02-668 Warsaw, Poland; current address, Electro Optics Technology Center, Halligan Hall, Tufts University, Medford, Massachusetts 02155.

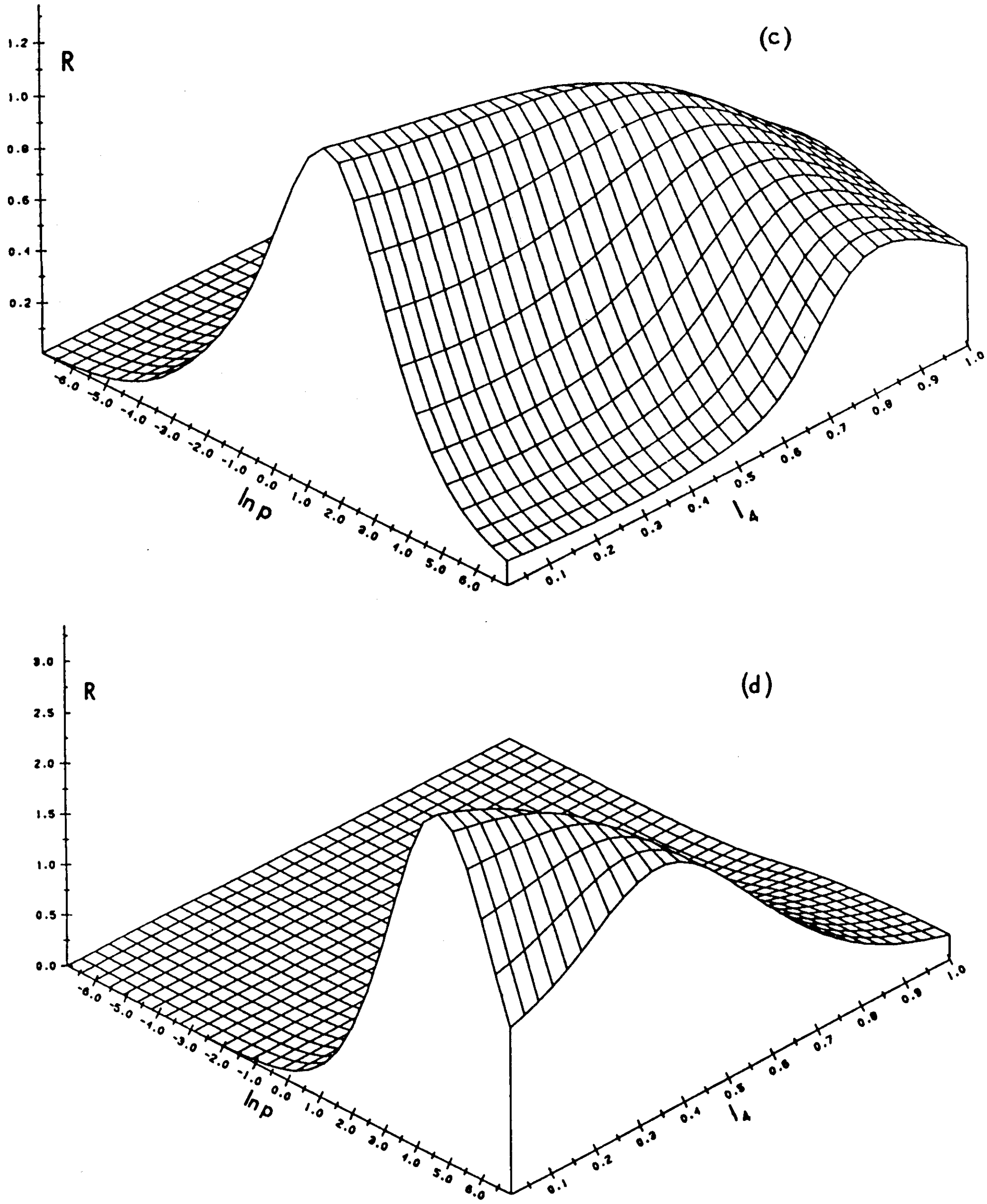


Fig. 9. PCR as a function of the pump ratio and the signal intensity: (a) $g_T = 10, g_R = g_P = g_S = 0, \alpha = 1$; (b) $g_T = 10, g_R = g_P = g_S = 0, \alpha = 4.15$; (c) $g_T = 10, g_R = g_S = 0, g_P = 5, \alpha = 1$; (d) $g_T = 10, g_R = g_S = 0, g_P = -5, \alpha = 1$. All numbers are given in units of inverse centimeters.

REFERENCES

1. M. C. Gower, J. Mod. Opt. **35,** 449 (1988).
2. P. Gunter, Phys. Rep. **93,** 199 (1982).
3. J. O. White, M. Cronin-Golomb, B. Fischer, and A. Yariv, Opt. Lett. **6,** 519 (1981).
4. M. Cronin-Golomb, B. Fischer, J. O. White, and A. Yariv, Opt. Lett. **7,** 313 (1982).
5. M. R. Belić, Opt. Lett. **12,** 105 (1987).
6. M. Cronin-Golomb, B. Fischer, J. O. White, and A. Yariv, IEEE J. Quantum Electron. **QE-20,** 12 (1985); M. Cronin-Golomb, J. Paslaski, and A. Yariv, Appl. Phys. Lett. **47,** 1131 (1985).
7. M. R. Belić, Phys. Rev. A, **37,** 1809 (1988).
8. J. Feinberg, Opt. Lett. **7,** 486 (1982).
9. J. Feinberg, J. Opt. Soc. Am. **72,** 46 (1982).
10. D. J. Gauthier, P. Narum, and R. W. Boyd, Phys. Rev. Lett. **58,** 1640 (1987).
11. W. Królikowski, Opt. Commun. **60,** 319 (1986); W. Królikowski and M. R. Belić, Opt. Lett. **13,** 149 (1988).
12. S. I. Stepanov, V. V. Kulikov, and M. P. Petrov, Opt. Commun. **44,** 19 (1982).
13. S. I. Stepanov and M. P. Petrov, Opt. Commun. **53,** 292 (1985).
14. Ph. Refregier, L. Solymar, H. Rajbenbach, and J. P. Huignard, J. Appl. Phys. **58,** 45 (1985).
15. Y. H. Ja, Opt. Quantum Electron. **15,** 529, 539 (1983).
16. P. D. Foote, T. J. Hal, N. B. Aldridge, and A. G. Levenston, Proc. IEEE **133,** 83 (1986).